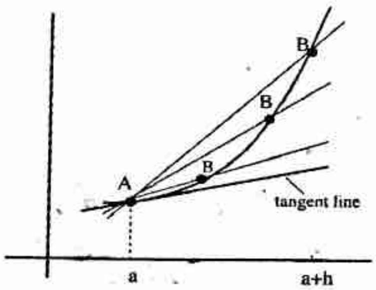
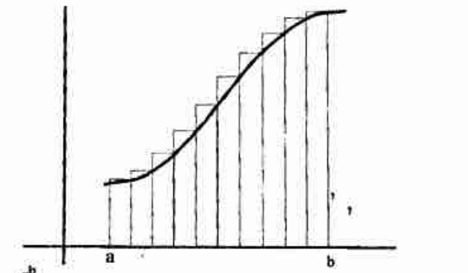
ADVANCED PLACEMENT MATHEMATICS

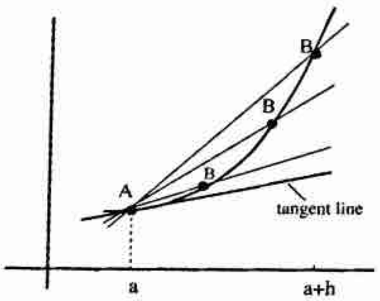
PREPARING FOR THE (BC) AP CALCULUS EXAMINATION



$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$



$\int_a^b f(x)\,dx = \lim_{n \to \infty} [f(x)\Delta x_1 + f(x)\Delta x_2 + f(x)\Delta x_3 + \dots + f(x)\Delta x_n]$



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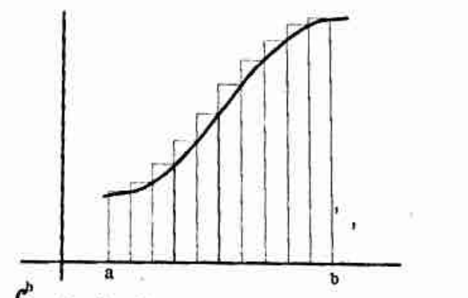

$\int_a^b f(x)\,dx = \lim_{n \to \infty} [f(x)\Delta x_1 + f(x)\Delta x_2 + f(x)\Delta x_3 + \dots + f(x)\Delta x_n]$

GEORGE W. BEST
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Index of Topics

The *multiple-choice* questions and the *free-response* questions are listed in separate sections, each of which is divided into four broad categories:

I. Continuity, Limits, Parametric, Polar & Vector Functions
II. Differentiation
III. Integration
IV. Series

MULTIPLE-CHOICE (Calculator-active problems are labeled C.)

Index of Topics

Index of Topics

Preface

Each student is expected to bring a graphing calculator to the Advanced Placement Calculus AB Examination. We have tried to make each of the six exams in this workbook as much like an actual AP Exam as possible. There are problems in which the student must decide whether to use the graphing calculator a lot, a little, or not at all. All of these objectives have been met and, at the same time, we included all of the topics in the BC Syllabus.

As in the College Board AP Course Description for Mathematics, our examinations are in two sections. Section I is all multiple-choice and takes one hour and 45 minutes. Section II is all free-response and takes one hour and 30 minutes. The two sections are given equal weight in the scoring.

1. Section I Part A (28 questions in 55 minutes). Calculators may not be used in this part of the exam.
2. Section I Part B (17 questions in 50 minutes). Calculators are allowed.
3. Section II Part A (2 questions in 30 minutes). Calculators are allowed.
4. Section II Part B (4 questions in 60 minutes). Calculators may not be used on this part of the exam.

In the calculator-active parts of the actual ETS Exams, Section I Part B and Section II above, approximately five multiple-choice and two free-response problems require the use of a graphing calculator. In order to provide a greater number of this type problem, our exams require the use of a calculator on about half the problems of Section I Part B and Section II.

We have also tried to create the problems in the spirit of *calculus reform*. Calculus reform implies a change in the mode of instruction as well as increased focus on concepts and less attention to symbolic manipulation; emphasis on modeling and applications; use of technology to explore and deepen understanding of concepts; projects and cooperative learning. We have included questions where functions are defined either graphically, numerically or symbolically in order to give the students more practice in different types of analysis.

Special thanks go to our wives Ann and Helen for their patience, understanding and encouragement.

In the hope of providing future students with a better workbook, the authors welcome your suggestions, corrections, problems of all sorts, and feedback in general. Please send your comments to:

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For The Student

There are six examinations in this workbook. Use them as suggested by your teacher, but about two weeks prior to the AP Exam you should try to find a three hour and thirty minute block of time to work through one entire exam. Each part of the exam should be carefully timed. Allow fifty-five minutes for Section I Part A, fifty minutes for Section I Part B, and ninety minutes for Section II. Take a ten minute break between Part A and Part B and also between Part B and Section II. This will give you a good measure of the topics that need more intensive review as well as give you a feel for the energy and enthusiasm needed on a three hour and fifteen minute exam. Repeat the above routine on a second exam four or five days before the AP to check your progress.

The questions on these exams are designed to be as much like the actual AP Exams as possible. However, we have included a greater percentage of medium level and difficult problems and fewer easy ones, in order to help you gain stamina and endurance. If you do a satisfactory job on these exams, then you should be confident of doing well on the actual AP Exam.

The answers to the multiple-choice questions and selected free-response questions are in the back of the workbook. A complete solution manual for all the problems is available from Venture Publishing. No matter how much of an exam you do at one sitting, we strongly urge you to check your answers when you are finished, not as you go along. You will build your confidence if you DO NOT use the "do a problem, check the answer, do a problem" routine.

The following is a list of common student errors:

1. If $f'(c) = 0$, then f has a local maximum or minimum at $x = c$.
2. If $f''(c) = 0$, then the graph of f has an inflection point at $x = c$.
3. If $f'(x) = g'(x)$, then $f(x) = g(x)$.
4. $\frac{d}{dx} f(y) = f'(y)$
5. Volume by washers is $\int_a^b (R-r)^2 dx$.
6. Not expressing answers in correct units when units are given.
7. Not providing adequate justification when justification is requested.
8. Wasting time erasing bad solutions. Simply cross out a bad solution after writing the correct solution.
9. Listing calculator results without the supporting mathematics. Recall that a calculator is to be used primarily to:
   a) graph functions,
   b) compute numerical approximations of a derivative and definite integral,
   c) solve equations.
10. Not answering the question that has been asked. For example, if asked to find the maximum value of a function, do not stop after finding the x-value where the maximum value occurs.

Table of Contents

EXAM I
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. $\displaystyle\int_0^5 \frac{dx}{\sqrt{3x+1}} =$

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 1

(D) 2

(E) 6

Ans ☐

---

2. Which of the following is continuous at $x = 0$?

I. $f(x) = |x|$

II. $f(x) = e^x$

III. $f(x) = \ln(e^x - 1)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) none

Ans ☐

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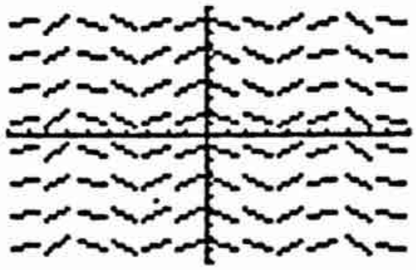
3. For what value of c will $x^2 + \frac{c}{x}$ have a relative minimum at $x = -1$?

(A) −4 (B) −2 (C) 2 (D) 4 (E) None of these

Ans

4. Which of the following could be a solution to the differential equation represented by the slope field at the right?

(A) $y = x^2$
(B) $y = \sin x$
(C) $y = \cos x$
(D) $y = e^x$
(E) $y = \ln x$



Ans

5. The volume of the solid generated by rotating about the x-axis the region enclosed between the curve $y = 3x^2$ and the line $y = 6x$ is given by

(A) $\pi\int_0^3 (6x - 3x^2)^2\,dx$

(B) $\pi\int_0^2 (6x - 3x^2)^2\,dx$

(C) $\pi\int_0^2 (9x^4 - 36x^2)\,dx$

(D) $\pi\int_0^2 (36x^2 - 9x^4)\,dx$

(E) $\pi\int_0^2 (6x - 3x^2)\,dx$

Ans

6. Suppose the function f is defined so that $f(0) = 1$ and its derivative, f', is given by $f'(x) = e^{\sin x}$. Which of the following statement are TRUE?

I $f''(0) = 1$

II The line $y = x + 1$ is tangent to the graph of f at $x = 0$.

III If $h(x) = f(x^3 - 1)$, then h is increasing for all real numbers x.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

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7. $\displaystyle\int_1^{\infty} \frac{3x^2}{(1 + x^3)^2}\,dx =$

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) nonexistent

Ans

---

8. The radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n \cdot 3^n}$ is

(A) 3

(B) 2

(C) 1

(D) 0

(E) ∞

Ans

9. There is a point between $P(1, 0)$ and $Q(e, 1)$ on the graph of $y = \ln x$ such that the tangent to the graph at that point is parallel to the line through points P and Q. The x-coordinate of this point is

(A) $e - 1$

(B) e

(C) -1

(D) $\frac{1}{e - 1}$

(E) $\frac{1}{e + 1}$

Ans

10. If $4x^2 + 2xy + 3y = 9$, then the value of $\frac{dy}{dx}$ at the point $(2, -1)$ is

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 2

(D) -2

(E) none of these

Ans

11. If $\frac{dy}{dx} = \sqrt{x}$, then the average rate of change of y with respect to x on the closed interval $[0, 4]$ is

(A) $\frac{1}{16}$ (B) 1 (C) $\frac{4}{3}$ (D) $\sqrt{2}$ (E) 2

Ans

12. A particle moves along the x-axis and its position for time $t \geq 0$ is $x(t) = \cos(2t) + \sec t$ When $t = \pi$, the acceleration of the particle is

(A) -6

(B) -5

(C) -4

(D) -3

(E) none of these

Ans

13. The region bounded by the x-axis and the part of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ is separated into two regions by the line $x = p$. If the area of the region for $0 \leq x \leq p$ exceeds the area of the region for $p \leq x \leq \pi$ by one square unit, then $p =$

(A) $\arccos \frac{1}{4}$

(B) $\arccos \frac{1}{3}$

(C) $\frac{\pi}{4}$

(D $\frac{\pi}{3}$

(E) $\frac{2\pi}{3}$

Ans

14. Let f be the function given by $f(x) = \ln x$. The third-degree Taylor polynomial for f about $x = 1$ is

(A) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3$

(B) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

(C) $(x-1) - \frac{1}{3}(x-1)^2 + \frac{1}{5}(x-1)^3$

(D) $(x-1) - \frac{1}{3}(x-1)^2 + \frac{1}{3}(x-1)^3$

(E) $(x-1) - (x-1)^2 + 2(x-1)^3$

Ans

15. If $h(x) = [f(x)]^2 + f(x)g(x)$, $f'(x) = g(x)$ and $g'(x) = -f(x)$, then $h'(x) =$

(A) $f(x)g(x)$
(B) $2f(x) - f(x)g(x)$
(C) $[f(x) + g(x)]^2$
(D) $[f(x) - g(x)]^2$
(E) $[g(x)]^2 + 2g(x)f(x) - [f(x)]^2$

Ans

16. Which of the following integrals gives the length of the graph of $y = \operatorname{Arcsin}\frac{x}{2}$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{1 - \frac{1}{\sqrt{4 - x^2}}}\, dx$

(B) $\int_a^b \sqrt{1 + \frac{1}{\sqrt{4 - x^2}}}\, dx$

(C) $\int_a^b \sqrt{1 - \frac{1}{4 - x^2}}\, dx$

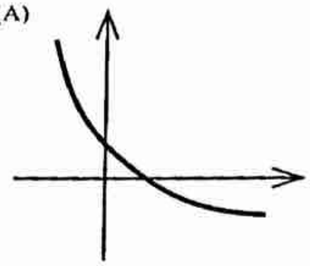
(D) $\int_a^b \sqrt{1 + \frac{1}{4 - x^2}}\, dx$

(E) $\int_a^b \left[1 + \frac{1}{4 - x^2}\right] dx$

Ans

17. If y is a function of x such that $\frac{dy}{dx} > 0$ for all x and $\frac{d^2y}{dx^2} < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

(A)



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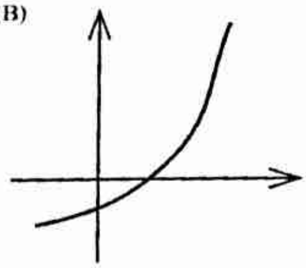

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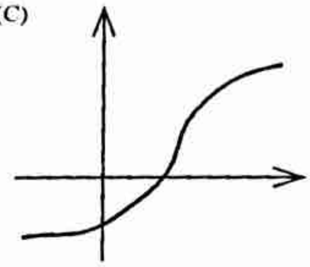

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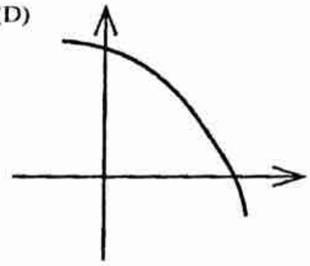
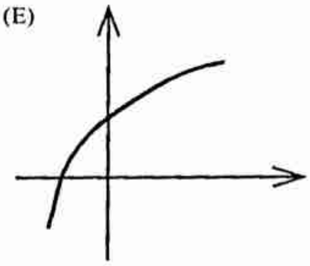


(E)



Ans

18. $\int \frac{1}{x^2 + x}\, dx =$

(A) $\frac{1}{2} \arctan\left(x + \frac{1}{2}\right) + C$

(B) $\ln\left|x^2 + x\right| + C$

(C) $\ln\left|\frac{x+1}{x}\right| + C$

(D) $\ln\left|\frac{x}{x+1}\right| + C$

(E) none of these

Ans

19. If $f(x) = x^2e^{-2x}$, then the graph of f is increasing for all x such that

(A) $0 < x < 1$ (B) $0 < x < \frac{1}{2}$ (C) $0 < x < 2$ (D) $x < 0$ (E) $x > 0$

Ans

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20. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about $x = 0$ for which of the following functions?

(A) $\sin x$

(B) $\cos x$

(C) e^x

(D) e^{-x}

(E) $\ln(1 + x)$

Ans

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21. Evaluate $\int_{-\pi/4}^{e-1} f(x)\,dx$ if $f(x) = \begin{cases} \sec^2 x & \text{for } x \le 0 \\ \frac{1}{x+1} & \text{for } x > 0 \end{cases}$

(A) 0
(B) 1
(C) 2
(D) e
(E) π

Ans

22. $\lim_{x \to 0} \left(1 + \frac{x}{2}\right)^{\cot x} =$

(A) $\frac{e}{2}$

(B) e

(C) $\sqrt{e}$

(D) 2e

(E) e^2

Ans

23. The area of the closed region bounded by the polar graph of $r = \sqrt{1 + \cos\theta}$ is given by

(A) $\int_0^{2\pi} \sqrt{1+\cos\theta}\, d\theta$

(B) $\int_0^{\pi} \sqrt{1+\cos\theta}\, d\theta$

(C) $2\int_0^{2\pi} (1+\cos\theta)\, d\theta$

(D) $\int_0^{\pi} (1+\cos\theta)\, d\theta$

(E) $2\int_0^{\pi} \sqrt{1+\cos\theta}\, d\theta$

Ans

24. $\lim_{h \to 0} \left[\frac{(3+h)^5 - 3^5}{9h} \right]$ is

(A) 0
(B) 1
(C) 45
(D) 405
(E) nonexistent

Ans

25.

x	0	4	7	9
$f(x)$	3	k	9	11

A function f is continuous on the closed interval [0, 9] and has values given in the table above. The trapezoidal approximation for $\int_0^9 f(t)\,dt$ found with 3 subintervals, indicated by the data in the table, is 57. What is the value of k?

(A) 9
(B) 7
(C) 6
(D) 5
(E) 2

Ans

26. $\int x\sqrt{1 - x^2}\, dx =$

(A) $\frac{(1-x^2)^{3/2}}{3} + C$

(B) $-(1-x^2)^{3/2} + C$

(C) $\frac{x^2(1-x^2)^{3/2}}{3} + C$

(D) $-\frac{x^2(1-x^2)^{3/2}}{3} + C$

(E) $-\frac{(1-x^2)^{3/2}}{3} + C$

Ans

27. Suppose a continuous function f and its derivative f' have values as given in the following table. Given that $f(1) = 2$, use Euler's method to approximate the value of $f(2)$.

x	1.0	1.5	2.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	2.0		

(A) 2.1 (B) 2.3 (C) 2.5 (D) 2.7 (E) 2.9

Ans

28. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y^2$, where $y(-1) = 1$?

(A) $y = \frac{1}{x}$ for $x \neq 0$

(B) $y = -\frac{1}{x}$ for $x < 0$

(C) $y = -\frac{1}{x}$ for $x > 0$

(D) $y = \frac{1}{x}$ for $x > 0$

(E) $y = \frac{1}{x}$ for $x < 0$

Ans

EXAM I
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

1. The slope of the curve $y = x^2 - e^{-x}$ at its point of inflection is

(A) $-\ln 2$ (B) $-\ln 4$ (C) $2 - \ln 4$ (D) $2 + \ln 4$ (E) $\frac{e^2}{2}$

Ans

2. If the graph of the parabola $y = 2x^2 + x + k$ is tangent to the line $3x + y = 1$, then $k =$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Ans

3. If a function f is defined by $f(x) = \int_0^x \frac{1}{1+t^4}\,dt$, which of the following statements are true?

I. $f(1) = \frac{1}{2}$

II. the graph of f is concave down at $x = 3$.

III. $f(x) + f(-x) = 0$ for all real numbers x.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

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4. Consider the function f defined on the domain $-0.5 \le x \le 0.5$ with $f(0) = 1$, and $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \sec^2(3x)$. Evaluate: $\int_0^{0.5} f(x)\,dx$.

(A) 0.294

(B) 0.794

(C) 1.294

(D) 1.794

(E) 4.700

Ans

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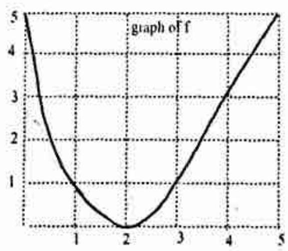
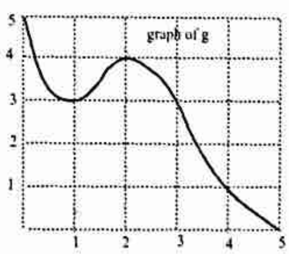
5. The graphs of functions f and g are shown at the right. The graph of g has a horizontal tangent at $x = 1$.
If $h(x) = f[g(x)]$, which of the following statements are true about the function h?





I. $h(2) = 5$.

II. h is increasing at $x = 4$.

III. The graph of h has a horizontal tangent at $x = 1$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

6. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \cos(t + \sqrt{t}\,)$. The total distance traveled by the particle from $t = 0$ to $t = 4$ is

(A) 0.481 (B) 1.069 (C) 1.449 (D) 1.932 (E) 2.416

Ans

7. The total area of the region bounded by the graphs of $y = \arctan x$ and $y = 4 - x^2$ is

(A) 10.802 (B) 10.972 (C) 11.142 (D) 11.31 (E) 11.482

Ans

8. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$ II. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\cos n}{3^n}$ III. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, III

Ans

9. If $x = t^2$ and $y = \ln(t^2 + 1)$, then at $t = 1$, $\frac{d^2y}{dx^2}$ is

(A) $-\frac{1}{4}$ (B) $-\frac{1}{2}$ (C) -1 (D) 0 (E) $\frac{1}{4}$

Ans

10. When a wholesale produce market has x crates of lettuce available on a given day, it charges p dollars per crate as determined by the supply equation $px - 20p - 6x + 40 = 0$. If the daily supply is decreasing at the rate of 8 crates per day, at what rate is the price changing when the supply is 100 crates?

(A) not changing

(B) increasing at $0.10 per day

(C) decreasing at $0.10 per day

(D) increasing at $1.00 per day

(E) decreasing at $1.00 per day

Ans

11. Let R be the region in the first quadrant bounded above by the graph of $f(x) = 2\text{Arc}\tan x$ and below by the graph of $y = x$. What is the volume of the solid generated when R is rotated about the x-axis?

(A) 1.217 (B) 2.276 (C) 2.693 (D) 6.666 (E) 7.151

Ans

12. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x = e^t$ and $y = e^{-t}$, then the speed of the particle at time $t = 1$ is

(A) 2.693 (B) 2.743 (C) 3.086 (D) 3.844 (E) 7.542

Ans

13. The level of air pollution at a distance x miles from a tire factory is given by

$$L(x) = e^{-0.1x} + \frac{1}{x^2}.$$

The average level of pollution between 15 and 25 miles from the factory is

(A) 0.144
(B) 0.156
(C) 0.162
(D) 0.168
(E) 0.250

Ans

14. What is the x-coordinate of the point on the curve $y = e^x$ that is closest to the origin?

(A) −0.452 (B) −0.426 (C) −0.400 (D) −0.374 (E) −0.372

Ans

15. Let $f(x) = e^{x/2}$. If the second-degree Taylor polynomial for f about $x = 0$ is used to approximate f on the interval $[0,2]$, what is the Lagrange error bound for the maximum error on the interval $[0,2]$?

(A) 0.028 (B) 0.113 (C) 0.453 (D) 0.499 (E) 0.517

Ans

16. If $\int f(x)\cdot\cos x\,dx = f(x)\cdot\sin x - \int 6x\cdot\sin x\,dx$, then $f(x)$ could be

(A) $-2x^3$
(B) $2x^3$
(C) $-3x^2$
(D) $3x^2$
(E) $x\sin x$

Ans

17. Which of the following statements are true?

I. If the graph of a function is always concave up, then the left-hand Riemann sums with the same subdivisions over the same interval are always less than the right-hand sums.

II. If the function f is continuous on the interval $[a, b]$ and $\int_a^b f(x)\,dx = 0$, then f must have at least one zero between a and b.

III. $f'(x) > 0$ for all x in an interval, then the function f is concave up in that interval.

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) none

Ans

EXAM I
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

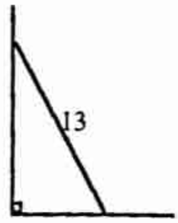
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. A 13 foot ladder is leaning against a wall so that the foot of the ladder is 1 foot from the wall. A gust of wind causes the ladder to begin sliding down the wall. The motion of the top of the ladder as it slides down the wall is described by

$y = -16t^2 + .05t + \sqrt{168}$, where t is measured in seconds.



(a) When does the top of the ladder reach the ground?

(b) Determine the velocity of the end of the ladder that is resting on the ground when it is 5 ft from the wall. Indicate units of measure.

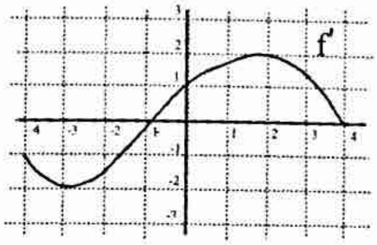
2. Suppose that $f(0) = 3$ and f' is the function shown below. Let $g(x) = (x^2 + 1)\cdot f(x)$.

(a) Evaluate $g'(0)$.

(b) Is g increasing, decreasing or neither at $x = 1$? Justify briefly.

(c) Estimate $g''(0)$.

(d) Is the graph of g concave up or concave down at $x = 1$? Justify briefly.

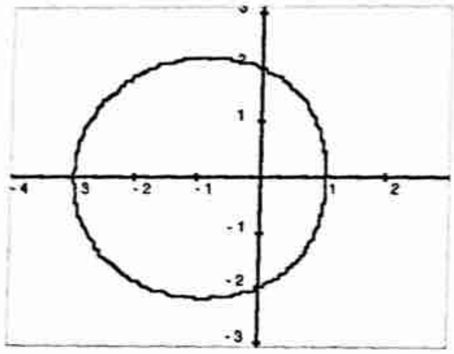


Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems

3. The graph of the polar curve $r = 2 - \cos\theta$ for $0 \le \theta \le 2\pi$. is shown in the figure.



(a) Write an integral expression for the area of the region inside the curve.

(b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Find $\frac{dy}{dx}$ as a function of θ.

(d) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the work that lead to your answer.

4. The Maclaurin Series for $f(x)$ is given by $\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \frac{x^6}{8!} + \ldots + \frac{(-1)^n x^{2n}}{(2n+2)!} + \ldots$

(a) For what values of x does the given series converge?

(b) Let $g'(x) = 1 - x^2 \cdot f(x)$. Write the Maclaurin series for $g'(x)$, showing the first three nonzero terms and the general term.

(c) Write $g'(x)$ in terms of a familiar function without using series. Then write $f(x)$ in terms of the same familiar function.

(d) Given that $g(0) = 3$ write $g(x)$ in terms of a familiar function without series.

5. A curve C is defined by the parametric equations $x = \dfrac{1}{\sqrt{t+1}}$ and $y = \dfrac{t}{t+1}$ for $t \geq 0$.

(a) Find $\dfrac{dy}{dx}$ in terms of t.

(b) Find an equation of the tangent line to C at $t = 3$.

(c) Set up but do not evaluate a definite integral representing the length of the curve C on the interval $0 \leq t \leq 1$.

(d) Find an equation for the curve C in terms of x and y.

6. A population is modeled by a function P that satisfies the differential equation $\frac{dP}{dt} = k(1200 - P)$, where t is time in years and k is the constant of proportionality. When t = 0, the population is 300, $P(0) = 300$.

(a) Find $P(t)$ in terms of t and k.

(b) If $P(4) = 600$, find the value of k.

(c) Find $\lim_{t \to \infty} P(t)$.

EXAM II
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. $\int_0^1 xe^{x^2}\,dx$

(A) $\frac{1}{2}(e-1)$ (B) $2(e-1)$ (C) $2e$ (D) e (E) $\frac{1}{2}e$

Ans ☐

---

2. If $f(x) = x^2 - 1$, then $\lim_{x \to 1} \frac{f(x+1) - f(2)}{x^2 - 1}$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) nonexistent

Ans ☐

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3. If $\cos x = e^y$ and $0 < x < \pi$, then $\dfrac{dy}{dx}$ is

(A) $-\tan x$
(B) $-\cot x$
(C) $\tan x$
(D) $\cot x$
(E) $\csc x$

Ans

4. If $y = \text{Arcsin}(e^{2x})$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$
(B) $\dfrac{e^{2x}}{\sqrt{1-e^{4x}}}$
(C) $\dfrac{2e^{2x}}{\sqrt{1+e^{4x}}}$
(D) $\dfrac{e^{2x}}{1-e^{4x}}$
(E) $\dfrac{2e^{2x}}{\sqrt{e^{4x}-1}}$

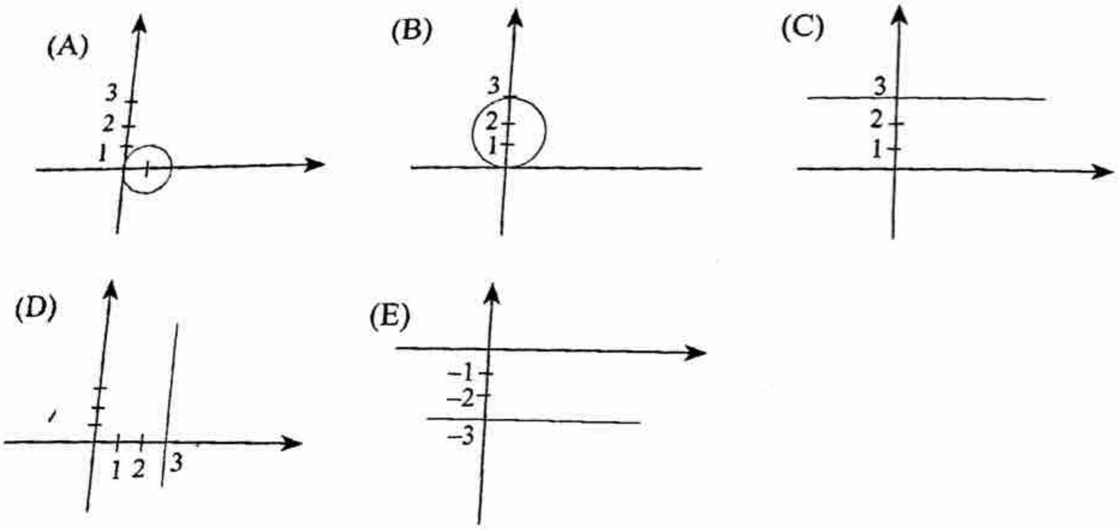
Ans

5. If $f(x) = \displaystyle\int_2^{2x} \frac{1}{\sqrt{t^3+1}}\, dt$, then $f'(1) =$

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\sqrt{2}$ (E) undefined

Ans

6. Which of the following represents the graph of the polar curve $r = 3\csc\theta$?



Ans

7. If $g(x) = \tan^2(e^x)$, then $g'(x) =$

(A) $2e^x \tan(e^x)\sec^2(e^x)$

(B) $2\tan(e^x)\sec^2(e^x)$

(C) $2\tan^2(e^x)\sec(e^x)$

(D) $e^x \sec^2(e^x)$

(E) $2e^x \tan(e^x)$

Ans

8. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

Ans

9. The coefficient of x^3 in the Taylor series for $f(x) = \int_0^x \frac{1+t}{e^t}\,dt$ about $x=0$ is

(A) -1 (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) 1

Ans

10. F and G are two functions whose derivatives exist for all real x; $F'(x) < 0$ and $G'(x) > 0$ for all real x. Which of the following could be true about the graphs of $y = F(x)$ and $y = G(x)$?

I. they do not intersect II. they intersect once III. they intersect more than once

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

11. The length of the curve determined by the parametric equations $x = \sin t$ and $y = t$ from $t = 0$ to $t = \pi$ is

(A) $\int_0^\pi \sqrt{\cos^2 t + 1}\ dt$

(B) $\int_0^\pi \sqrt{\sin^2 t + 1}\ dt$

(C) $\int_0^\pi \sqrt{\cos t + 1}\ dt$

(D) $\int_0^\pi \sqrt{\sin t + 1}\ dt$

(E) $\int_0^\pi \sqrt{1 - \cos t}\ dt$

Ans

12. Let the piecewise function f be defined by $f(x) = \begin{cases} \dfrac{\cos(2x) - 1}{x^2}, & \text{for } x \neq 0 \\ m, & \text{for } x = 0 \end{cases}$.

Determine a value for m so that the function f is continuous at $x = 0$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

13. The slope of the line tangent to the graph of $\ln(x + y) = x^2$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) $e - 1$ (D) $2e - 1$ (E) $e - 2$

Ans

14. At $x = 0$, which of the following is true of the function $f(x) = \sin x + e^{-x}$?

(A) f is increasing
(B) f is decreasing
(C) f is discontinuous
(D) The graph of f is concave up
(E) The graph of f is concave down

Ans

15. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \cdot \frac{(x-3)^n}{2^n}$ is

(A) 4
(B) 3
(C) 2
(D) 1
(E) 0

Ans

16. A particle moves along the curve $x^2y = 2$ at time $t > 0$. If $\frac{dy}{dt} = 8$ when $x = -1$, what is the value of $\frac{dx}{dt}$ at that time?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

17. If $\int_a^b f(x)\,dx = 3$ and $\int_a^b g(x)\,dx = 2$, which of the following must be true?

I. $f(x) > g(x)$ for all $a \le x \le b$

II. $\int_a^b [f(x) - g(x)]\,dx = 1$

III. $\int_a^b [f(x) \cdot g(x)]\,dx = 6$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

Ans

18. Consider the curve in the xy-plane represented by $x = \frac{2}{t}$ and $y = \ln t$ for $t > 0$. The slope of the line tangent to the curve at the point where $x = 1$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

Ans

19. If $\frac{dy}{dx} = xy^2$, then y could be

(A) $\frac{-1}{x^2 + 1}$

(B) $-\frac{1}{x^2} + 1$

(C) $\frac{-2}{x^2 + 1}$

(D) $3e^{x^2/2}$

(E) $3e^{x^2/2} + 1$

Ans

20. $\int \frac{x}{x + 2}\, dx$

(A) $x \ln|x+2| + C$

(B) $x + 2 \ln|x+2| + C$

(C) $x - 2 \ln|x+2| + C$

(D) $x - \ln|x+2| + C$

(E) $x - \text{Arctan}\, x + C$

Ans

21. Let f be a function with $f(2) = 4$ and derivative $f'(x) = \sqrt{x^3 + 1}$. Using a tangent line approximation to the graph of f at $x = 2$, estimate $f(2.2)$.

(A) 4.0 (B) 4.2 (C) 4.4 (D) 4.6 (E) 4.8

Ans

22. A region in the plane is bounded by $y = \dfrac{1}{\sqrt{x}}$, the x-axis, the line $x = m$ and the line $x = 2m$ where $m > 0$. A solid is formed by revolving the region about the x-axis. The volume of this solid

(A) is independent of m

(B) increases as m increases

(C) decreases as m increases

(D) increases until $m = \frac{1}{2}$, then decreases

(E) is none of the above

Ans

23. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left\langle \sin(3t - \frac{\pi}{2}),\ 3t^2 \right\rangle$, then at time $t = \frac{\pi}{2}$ the velocity vector is

(A) $\langle -3, 3\pi \rangle$ (B) $\langle -1, 3\pi \rangle$ (C) $\langle -1, 2\pi \rangle$ (D) $\langle 3, 2\pi \rangle$ (E) $\langle 3, 3\pi \rangle$

Ans

24. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n+1}{2n+1}$ II. $\sum_{n=1}^{\infty} \frac{1}{4n^2}$ III. $\sum_{n=1}^{\infty} \frac{2^n}{n3^n}$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

Ans

25. F and G are differentiable functions such that $F(x) = \int_0^x G(t)\,dt$. If $F(a) = 3$ and $F(b) = 3$, where $0 < a < b$, which of the following must be true?

(A) $G(x) = 0$ for some x such that $a < x < b$

(B) $G(x) = 0$ for all x such that $a < x < b$

(C) $G(x) > 0$ for all x such that $a < x < b$

(D) $F(x) \geq 0$ for all x such that$a < x < b$

(E) $F(x) = 0$ for some x such that $a < x < b$

Ans

26. $\int_0^1 xe^{-x}\,dx =$

(A) 1 (B) $1 - \frac{2}{e}$ (C) $\frac{2}{e} - 1$ (D) $1 + \frac{2}{e}$ (E) $-\frac{2}{e}$

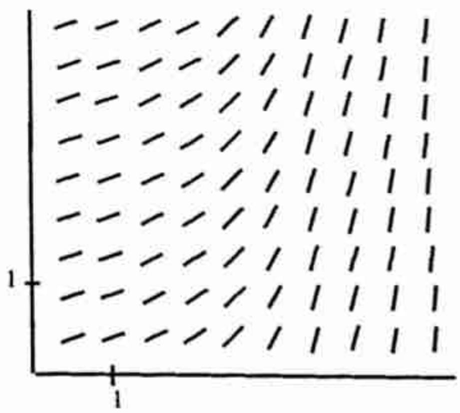
Ans

27. If the average rate of change of a function f over the interval from $x = 2$ to $x = 2 + h$ is given by $7e^h - 4\cos(2h)$, then $f'(2) =$

(A) –1 (B) 0 (C) 1 (D) 2 (E) 3

Ans

28. The slope field shown in the figure at the right represents solutions to a certain differential equation. Which of the following could be a specific solution to that differential equation ?



(A) $y = e^{-x}$

(B) $y = \sin x$

(C) $y = \sqrt{x}$

(D) $y = \ln x$

(E) $y = e^{0.5x}$

Ans

EXAM II
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. Let $f(x) = (1+x)^{1/3}$. The Taylor polynomial of degree 2 for the function f about $x = 0$ is

(A) $1+x-x^2$ (B) $1+\frac{x}{3}+\frac{x^2}{9}$ (C) $1+\frac{x}{3}-\frac{x^2}{9}$

(D) $1+\frac{x}{3}+\frac{2x^2}{9}$ (E) None of the above.

Ans

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2. Which of the following are true about the function f if its derivative is defined by

$$f'(x) = (x-1)^2(4-x)\ ?$$

I. f is decreasing for all $x < 4$.

II. f has a local maximum at $x = 1$.

III. The graph of f is concave up for all $1 < x < 3$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II and III

Ans

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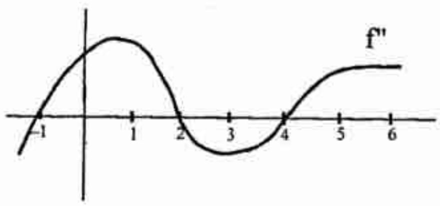
3. A tangent line drawn to the graph of $y = \dfrac{4x}{1 + x^3}$ at the point $(1, 2)$ forms a right triangle with the coordinate axes. The area of the triangle is

(A) 3.0
(B) 3.5
(C) 4.0
(D) 4.5
(E) 5.0

Ans

4. The graph of the second derivative f'' for a function f is shown below.



If f is increasing at $x = -1$, which of the following statements must be true?

I. $f'(2) = f'(4)$ II. $f'(4) > f'(-1)$ III. $f'(4) > 0$

(A) I only (B) II only (C II and III only (D) I and III only (E) I, II and III

Ans

5. Suppose a particle is moving along a coordinate line and its position at time t is given by $s(t) = \dfrac{9t^2}{t^2 + 2}$. For what value of t in the interval $[1, 4]$ is the instantaneous velocity equal to the average velocity?

(A) 2.199 (B) 2.209 (C) 2.219 (D) 2.229 (E) 2.239

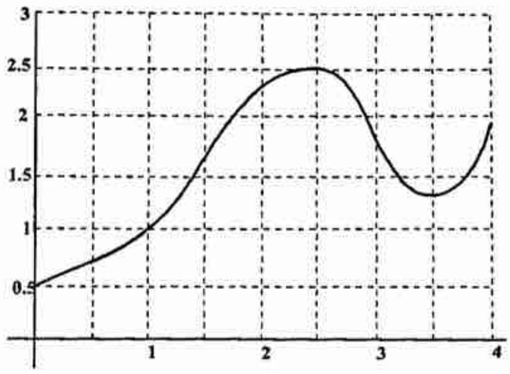
Ans

6. If $f(x) = \dfrac{e^x}{x+1}$ and $g(x) = \dfrac{x}{x+1}$, then $f'(x) = g'(x)$ at $x =$

(A) 0.563 (B) 0.565 (C) 0.567 (D) 0.569 (E) 0.571

Ans

7. A graph of the function f is shown at the right. Which of the following statements are true?



I. $f(1) > f'(3)$

II. $\int_1^2 f(x)\,dx > f'(3.5)$

III. $\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h} > \dfrac{f(2.5) - f(2)}{2.5 - 2}$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II and III

Ans

8. Which of the following three improper integrals converge?

I. $\int_0^\infty \dfrac{1}{1+x^2}\,dx$ II. $\int_1^\infty \dfrac{1}{x^2}\,dx$ III. $\int_0^1 \dfrac{1}{x}\,dx$

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III

Ans

9. The proportion of students that have heard a rumor at time t is modeled by the function P that satisfies the logistics differential equation $\frac{dP}{dt} = 3P(3-2P)$. What proportion of the student population has heard the rumor when it is spreading the fastest?

(A) 25% (B) 40% (C) 50% (D) 60% (E) 75%

Ans

10. If the graph of $y = f(x)$ contains the point (0, 1), and if $\frac{dy}{dx} = \frac{x\sin(x^2)}{y}$, then $f(x)$=

(A) $\sqrt{2-\cos(x^2)}$

(B) $\sqrt{2} - \cos(x^2)$

(C) $2 - \cos(x^2)$

(D) $\cos(x^2)$

(E) $\sqrt{2-\cos x}$

Ans

11. If $g(x) = \int_0^{x^2} (t^2 + 7)^{2/3}\,dt$, then $g''(1) =$

(A) $\frac{2}{3}$ (B) 4 (C) $\frac{16}{3}$ (D) $\frac{32}{3}$ (E) 8

Ans

12. The graph of the function f represented by the Maclaurin series $1+2x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+\frac{16x^4}{4!}+\ldots$ intersects the graph of $y=2-x^3$ at the point where $x=$

(A) 0.182 (B) 0.337 (C) 0.587 (D) 0.799 (D) 0.989

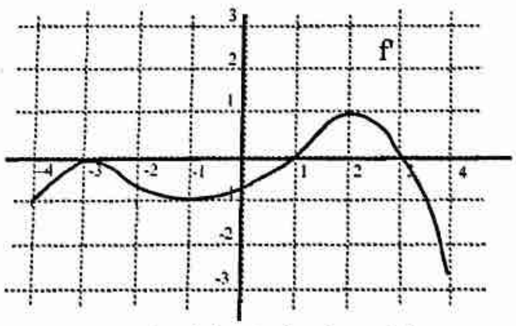
Ans

13. The figure at the right shows the graph of f', the *derivative* of a function f. The domain of f is the interval $-4 \le x \le 4$. Which of the following must be true about the graph of f?

I. At the points where $x=-3$ and $x=2$ there are horizontal tangents.

II. At the point where $x=1$ there is a relative minimum point.

III. At the point where $x=-3$ there is an inflection point.



graph of the derivative of f

(A) None (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

14. The volume of the solid generated by revolving the first quadrant region bounded by the curve $y = e^{x/2}$ and the lines $x = \ln 3$ and $y = 1$ about the x-axis is

(A) 2.802 (B) 2.832 (C) 2.862 (D) 2.892 (E) 2.922

Ans

15. Let f be a function such that f and all its derivatives are continuous and the third derivative of f satisfies the inequality $|f'''(x) \le 10|$. If a second degree polynomial for f about $x = 1$ is used to approximate $f(1.4)$, what is the corresponding Lagrange Error Bound?

(A) 0.027 (B) 0.107 (C) 0.213 (D) 0.533 (E) none of these

Ans

16. If the function f is differentiable on the interval $[a, b]$ and $a < c < b$, which of the following statements are true?

I. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

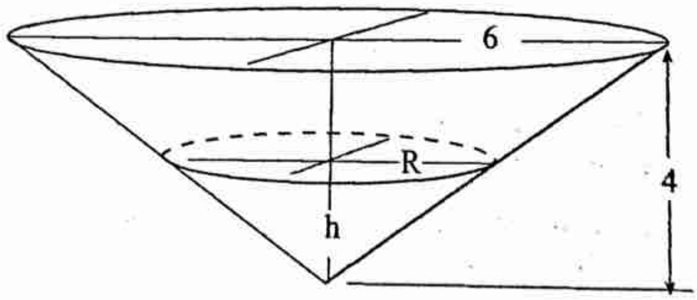
II. There exists a number d in (a, b) such that $f'(d) = \dfrac{f(b) - f(a)}{b - a}$

III. $\lim_{x \to c} f(x) = f(c)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

17. The conical reservoir shown at the right has diameter 12 feet and height 4 feet. Water is flowing into the reservoir at the constant rate of 10 cubic feet per minute. At the instant when the surface of the water is 2 feet above the vertex, the water level is rising at the rate of



$$V = \frac{1}{3}\pi R^2 h$$

(A) 0.177 ft/min

(B) 0.354 ft/min

(C) 0.531 ft/min

(D) 0.708 ft/min

(E) 0.885 ft/min

Ans

EXAM II
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. A particle moves along a line in such a way that at time t, $1 \le t \le 8$, its position is given by

$$s(t) = \int_1^t [1 - x(\cos x) - (\ln x)(\sin x)]\, dx.$$

(a) Write a formula for the velocity of the particle at time t.

(b) At what instant does the particle reach it maximum speed?

(c) When is the particle moving to the left?

(d) Find the total distance traveled by the particle from $t = 1$ to $t = 8$.

2. The following table represents the diameter (feet) of the cross section of a tree at various heights (feet) above the ground. Assume that each cross section is circular.

Height (ft.)	2	6	10	14	18	22	26	30
Diameter (ft.)	2.0	2.0	2.0	1.8	1.6	1.5	1.3	1.2

(a) Approximate how fast the diameter of the tree is changing 22 ft. above the ground. Indicate units of measure.

(b) Use the trapezoid rule to approximate the volume of the tree from 14 ft to 30 ft above the ground. Indicate units of measure.

(c) The section of the tree from 2 feet to 8 feet is used to make a rectangular beam of length 6 feet. The strength of the beam varies jointly as its width and the square of its height. What should be the width and height of the beam in order to have the strongest beam?

Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems

3. Let functions f and g be defined by $f(x) = x$ and $g(x) = x + \frac{k}{x}$, where k is a positive constant.

(a) Find the average value of g on the interval $[1, 3]$ if $k = 2$.

(b) If R is the region between the graphs of f and g on the interval $[1, 3]$, find in terms of k, the volume of the solid generated when R is rotated about the x-axis.

(c) Set up but <u>do not</u> evaluate an integral expression in terms of k for the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

4. Let f be a differentiable function such that f'' is continuous and f and f' have the values given in the table below.

x	0	1	2	3	4	5
$f(x)$	1	17	3	8	9	11
$f'(x)$	25	21	19	15	13	–2

Use the information in the table to

(a) approximate $f''(x)$ at $x = 2$.

(b) evaluate $\int_0^2 xf'(x^2)\,dx$

(c) evaluate $\int_1^3 xf''(x)\,dx$

5. Let f be the function defined by $f(x) = xe^{-kx}$, where k is a positive constant.

(a) Find, in terms of k, the x-coordinate of each critical point of f.

(b) For each critical number x, determine whether $f(x)$ is a relative maximum, relative minimum, or neither. Justify your answer.

(c) On what interval(s) is the graph of f concave up?

(d) Write an equation of the horizontal asymptote for the graph of f.

6. Let f be the function defined by $f(x) = \ln(x+1)$.

(a) Find $f^{(n)}(0)$ for $n = 1$ to $n = 3$, where $f^{(n)}$ is the n^{th} derivative of f.

(b) Write the first three nonzero terms and the general term for the Taylor series expansion of $f(x)$ about $x = 0$.

(c) Determine the radius of convergence for the series in part (b). Show your reasoning.

(d) Use the series in part (b) to evaluate $\int_0^{0.5} f(x)\,dx$ with an error no greater than 0.01.

EXAM III
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. A particle moves on the x-axis in such a way that its position at time t, $t > 0$, is given by $x(t) = (\ln t)^2$. At what value of t does the velocity of the particle attain its maximum?

(A) 1 (B) $e^{1/2}$ (C) e (D) $e^{3/2}$ (E) e^2

Ans ☐

---

2. Which of the following is equal to $\int_0^{\pi} \cos x\, dx$?

(A) $\int_0^{\pi} \sin x\, dx$ (B) $\int_{-\pi/2}^{\pi/2} \cos x\, dx$ (C) $\int_{-\pi/2}^{\pi/2} \sin x\, dx$

(D) $\int_{\pi}^{2\pi} \sin x\, dx$ (E) $\int_{\pi/2}^{3\pi/2} \cos x\, dx$

Ans ☐

---

3. A solution to $\dfrac{dy}{dx} = \dfrac{1}{xy}$ that goes through the point (1, 1) is

(A) $\dfrac{1}{x^2}$

(B) $\sqrt{2 \ln x} + 1$

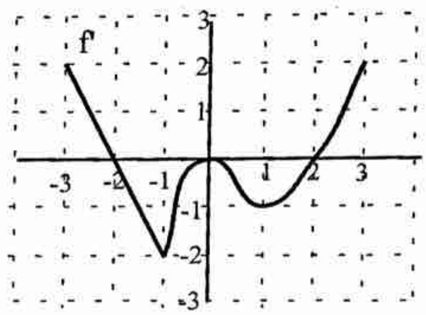
(C) $\sqrt{2 \ln x + 1}$

(D) $\sqrt{\ln x + 1}$

(E) e^{x-1}

Ans

4. At the right is the graph of $y = f'(x)$, the derivative of $y = f(x)$. The domain of f is the interval $-3 \le x \le 3$. Which of the following must be true about the graph of f?



I. f is increasing on $-3 < x < -2$.

II. The graph of f is concave down on $-3 < x < -1$.

III. The maximum value of $f(x)$ on the interval $-3 < x < 2$ is $f(-3)$.

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

5. $\displaystyle\int_e^{+\infty} \frac{1}{x(\ln x)^2}\, dx$

(A) $\frac{1}{e}$ (B) $-\frac{1}{e}$ (C) e (D) 1 (E) divergent

Ans

6. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ is 1. What is the interval of convergence ?

(A) $-1 < x < 1$

(B) $-3 < x < 1$

(C) $1 < x < 3$

(D) $1 < x \le 3$

(E) $1 \le x < 3$

Ans

7. Suppose a population of bears grows according to the logistic differential equation

$$\frac{dP}{dt} = 2P - 0.01P^2$$

where P is the number of bears at time t in years. Which of the following statements are true?

I. The growth rate of the bear population is greatest at $P = 100$.

II. If $P > 200$, the population of bears is decreasing.

III. $\lim_{t \to \infty} P(t) = 200$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Ans

8. The substitution of $x = \sin\theta$ in the integral $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$, results in

(A) $\int_0^{1/2} \frac{\sin^2\theta}{\cos\theta}\,d\theta$ (B) $\int_0^{1/2} \sin^2\theta\,d\theta$ (C) $\int_0^{\pi/6} \sin^2\theta\,d\theta$

(D) $\int_0^{\pi/3} \sin^2\theta\,d\theta$ (E) $\int_0^{1/2} \frac{\cos^2\theta}{\sin\theta}\,d\theta$

Ans

---

9. Let f and g be functions whose derivatives exist for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} f'(x) = 6$ and $\lim_{x\to 0} g'(x) = 2$, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) 1

(C) 3

(D) $\frac{f'(x)}{g'(x)}$

(E) nonexistent

Ans

---

10. The slope of the tangent line to the graph of $y = \text{Arctan}\,\frac{x}{2}$ at the point $\left(2, \frac{\pi}{4}\right)$ is

(A) $\frac{1}{16}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

Ans

11. If $\frac{dy}{dx} = \frac{3 \sin x}{\sec^2 x}$, then $y =$

(A) $\ln|\cos x| + C$
(B) $\sec x + C$
(C) $\cos^3 x + C$
(D) $-3\cos^3 x + C$
(E) $-\cos^3 x + C$

Ans

12. Consider the set of all right circular cylinders for which the sum of the height and the diameter is 18 inches. What is the radius of the cylinder with the maximum volume?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Ans

13. The total area of the region enclosed by the polar graph of $r = 1 + \sin\theta$ is

(A) $\frac{\pi}{2}$
(B) π
(C) $\frac{3\pi}{2}$
(D) 2π
(E) $\frac{5\pi}{2}$

Ans

14. The acceleration of a particle moving along the x-axis at any time $t \geq 0$ is given by $a(t) = 1 + e^{-t}$. At $t = 0$ the velocity of the particle is -2 and its position is 3. The position of the particle at any time t is

(A) $\frac{t^2}{2} - t + e^{-t} + 2$

(B) $\frac{t^2}{2} - 3t + e^{-t} + 2$

(C) $\frac{t^2}{2} - t - e^{-t} + 2$

(D) $\frac{t^2}{2} - 3t - e^{-t} + 2$

(E) $t^2 - t + e^{-t} + 2$

Ans

15. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = \text{a}$ and $x = \text{b}$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\, dx$

(B) $\int_a^b \sqrt{x + \tan x}\, dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\, dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\, dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\, dx$

Ans

16. If $v = \sin(u^2 - 1)$ and $u = \sqrt{x^2 + 1}$, then $\dfrac{dv}{dx}$ is

(A) $\dfrac{\cos(x^2)}{2\sqrt{x^2+1}}$

(B) $\dfrac{x\cos(x^2)}{2\sqrt{x^2+1}}$

(C) $\dfrac{x\cos(x^2-1)}{\sqrt{x^2+1}}$

(D) $2x\cos(x^2)$

(E) $\cos(x^2)$

Ans

17. The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x\to c} f(x)$ exists (B) $\lim_{x\to c} f(x) = f(c)$ (C) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

(D) $f(c)$ is defined (E) $f'(c)$ exists

Ans

18. The area of the region in the first quadrant under the curve $y = \dfrac{1}{\sqrt{1-x^2}}$ bounded on the left by $x = \frac{1}{2}$ and on the right by $x = 1$ is

(A) ∞

(B) π

(C) $\dfrac{\pi}{2}$

(D) $\dfrac{\pi}{3}$

(E) none of these

Ans

19. The function f is defined by $f(x) = 3x^2 - x^3 + h$. For which values of h will f have three distinct zeros ?

(A) all $h > 4$
(B) $0 < h < 4$
(C) all $h < 0$
(D) $-4 < h < 0$
(E) all $h < -4$

Ans

---

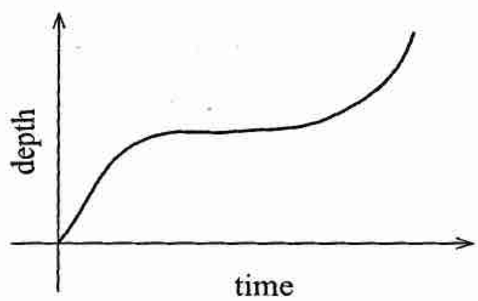
20. Let f be a function that has derivatives of all order for all real numbers. Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$ and $f'''(0) = 4$. Then the coefficient of x^3 in the Taylor series for f about $x = 0$ is

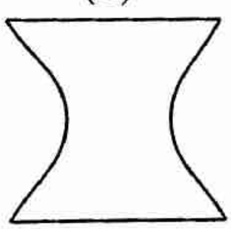
(A) -3 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1 (D) 4

Ans

---

21. Every cross section perpendicular to the axis of a container is a circle. Water is flowing into the container at a constant rate. A graph of the depth of the water as a function of time is shown at the right. Which of the following best describes the profile of the container?



(A)

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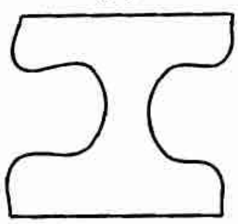
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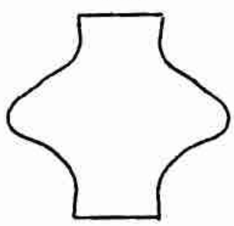
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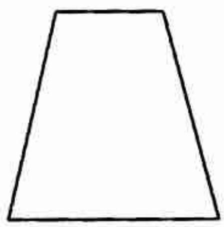
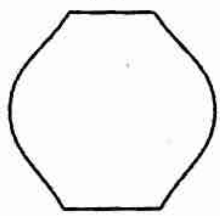

(E)


Ans

22. The sales of a small company are expected to grow at a rate given by $\frac{dS}{dt} = 300t + t^{1/2} + t^{3/2}$, where $S(t)$ is the sales in dollars in t days. The accumulated sales from the first day through the fourth day is approximately

(A) \$2400
(B) \$2406
(C) \$2412
(D) \$2418
(E) \$2424

Ans

23. If $F(x) = \int_{\pi/2}^{x} 4t \sin\left(\frac{t}{3}\right) dt$, then an equation of the line tangent to $y = F(x)$ at the point where $x = \frac{\pi}{2}$ is

(A) $2x - \pi y - \pi = 0$
(B) $2x - 2y - \pi = 0$
(C) $2\pi x - 2y - \pi^2 = 0$
(D) $\pi x - 2y - \pi^2 = 0$
(E) $\pi x - y - \pi = 0$

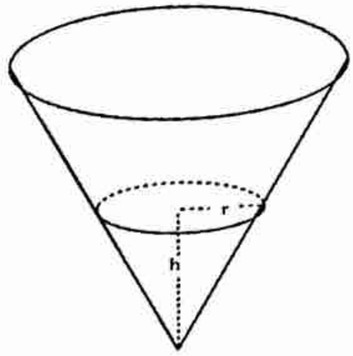
Ans

24. If $\int_0^k \frac{\sec^2 x}{1 + \tan x}\, dx = \ln 2$, then the value of k is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) π

Ans

25. A conical tank is being filled with water at the rate of 16 ft^3/min. The rate of change of the depth of the water is 4 times the rate of change of the radius of the water surface. At the moment when the depth is 8 ft and the radius of the surface is 2 ft, the area of the surface is changing at the rate of



Volume of Cone = $\frac{1}{3}\pi r^2 h$

(A) $\frac{1}{\pi}$ ft^2/min

(B) 1 ft^2/min

(C) 4 ft^2/min

(D) 4π ft^2/min

(E) 16π ft^2/min

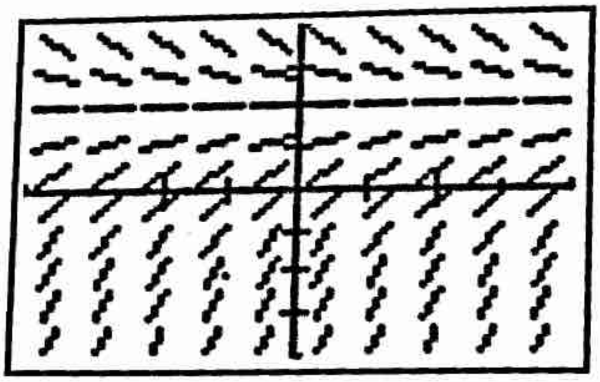
Ans

26. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+1}$ and $y(0) = 0$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{9}{10}$

Ans

27. A slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given at the right. Which of the following could be a solution ?



(A) $y = 2 + \ln x$

(B) $y = 2 - \ln x$

(C) $y = 2 - e^{x}$

(D) $y = 2 - e^{-x}$

(E) $y = 2 + e^{2x}$

Ans

28. $\int x\,e^{2x}\,dx =$

(A) $\frac{1}{4}\,e^{2x}(2x-1)+C$

(B) $\frac{1}{2}e^{2x}(2x-1)+C$

(C) $\frac{1}{4}e^{2x}(4x-1)+C$

(D) $\frac{1}{2}e^{2x}(x-1)+C$

(E) $\frac{1}{4}\,e^{2x}(x-1)+C$

Ans

EXAM III
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. Which of the following must be true about a particle that starts at $t = 0$ and moves along a number line if its position at time t is given by $s(t) = (t-2)^3(t-6)$?

I. The particle is moving to the right for $t > 5$.
II. The particle is at rest at $t = 2$ and $t = 6$.
III. The particle changes direction at $t = 2$.

(A) I only (B) II only (C) III only (D) I and III only (E) none

Ans

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2. The approximate *average* rate of change of the function $f(x) = \int_0^x \sin(t^2)\,dt$ over the interval $[1, 3]$ is

(A) 0.155 (B) 0.232 (C) 0.309 (D) 0.386 (E) 0.463

Ans

---

3. $\int \frac{1}{\sqrt{x}\,(1-\sqrt{x})}\,dx =$

(A) $\frac{1}{2}\ln\left|1-\sqrt{x}\right|+C$

(B) $2\ln\left|1-\sqrt{x}\right|+C$

(C) $4\sqrt{1-\sqrt{x}}\;+C$

(D) $-2\ln\left|1-\sqrt{x}\right|+C$

(E) none of these

Ans

4. Let R be the region in the first quadrant that is enclosed by the graph of $f(x)=\ln(x+1)$, the x-axis and the line $x=\text{e}$. What is the volume of the solid generated when R is rotated about the line $y=-1$?

(A) 5.037 (B) 6.545 (C) 10.073 (D) 20.146 (E) 28.686

Ans

5. $\lim\limits_{h\to 0}\dfrac{\int_1^{1+h}\sqrt{x^3+8}\;dx}{h}$ is

(A) 0

(B) 1

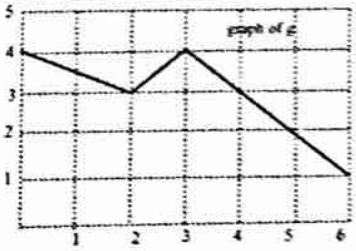
(C) 3

(D) $2\sqrt{2}$

(E) nonexistent

Ans

6. A graph of the function g is shown in the figure. If the function h is defined by $h(x) = g(x^2)$, use the graph to estimate $h'(2)$.



(A) –8 (B) –4 (C) –2 (D) 2 (E) 4

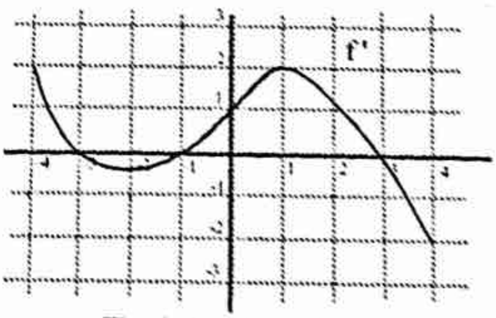
Ans

7. Let f be a function having 5 derivatives on the interval [2, 2.9] and assume that $|f^{(5)}(x)| \le 0.8$ for all x in the interval [2, 2.9]. If the fourth-degree Taylor polynomial for f about $x = 2$ is used to approximate f on the interval [2, 2.9], what is the Lagrange error bound for the maximum error on the interval [2, 2.9]?

(A) 0.004 (B) 0.011 (C) 0.022 (D) 0.033 (E) 0.044

Ans

8. A function f is defined on the closed interval $-4 \le x \le 4$. The graph of f', the derivative of f, is shown at the right. If the graph of f' has horizontal tangents at $x = -2$ and $x = 1$, which of the following must be true about the original function f?



The derivative of f

I. f is increasing on the interval (–2, 1).

II. f is continuous at $x = 0$.

III. The graph of f has an inflection point at $x = -2$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

9. A curve is defined parametrically by $x = e^t$ and $y = 2e^{-t}$. An equation of the tangent line to the curve at $t = \ln 2$ is

(A) $x - 2y + 3 = 0$

(B) $x + 2y - 4 = 0$

(C) $x + 2y - 5 = 0$

(D) $x - 2y - 4 = 0$

(E) $2x + y - 5 = 0$

Ans

10. If $x^2 - y^2 = 25$ then $\dfrac{d^2y}{dx^2} =$

(A) $-\dfrac{x}{y}$ (B) $\dfrac{5}{y^2}$ (C) $-\dfrac{x^2}{y^3}$ (D) $-\dfrac{25}{y^3}$ (E) $\dfrac{4}{y^3}$

Ans

11. Which of the following series are convergent?

I. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} + \ldots$

II. $1 - \frac{1}{2} + \frac{1}{3} - \ldots + \frac{(-1)^n}{n} + \ldots$

III. $2 + 1 + \frac{8}{9} + \ldots + \frac{2^n}{n^2} + \ldots$

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II and III

Ans

12. If $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \frac{x^2+1}{x^2}$, then $g(x)$ could be equal to

(A) x^{-3} (B) $-2x^{-3}$ (C) $\frac{x^2-1}{x}$ (D) $x - x^2$ (E) $1 + x^{-2}$

Ans

13. Two particles move along the x-axis and their positions at time $0 \le t \le 2\pi$ are given by $x_1 = \cos t$ and $x_2 = e^{(t-3)/2} - 0.75$. For how many values of t do the two particles have the same velocity?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Ans

14. If a left Riemann sum overapproximates the definite integral $\int_0^4 f(x)\,dx$ and a trapezoid sum underapproximates the integral $\int_0^4 f(x)\,dx$, which of the following could be a graph of $y = f(x)$?

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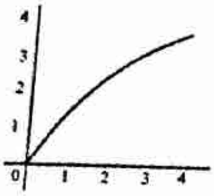

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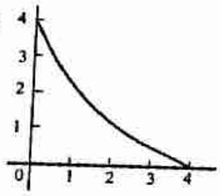

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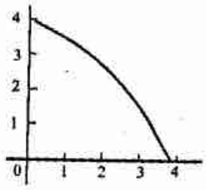

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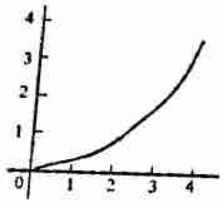
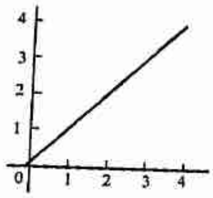


(E)


Ans

15. The radius of convergence of the series $x + \frac{2x^2}{2^2} + \frac{6x^3}{3^3} + \cdots + \frac{n!x^n}{n^n} + \cdots$ is

(A) ∞ (B) e^2 (C) e (D) $\frac{e}{2}$ (E) 0

Ans

16. Let f be a function whose graph is concave down on the the closed interval $[1, 2]$, with selected values shown in the table below.

x	1.1	1.3	1.5	1.7	1.9
$f(x)$	12	18	21	23	24

If f' and f'' are defined for all x in $[1, 2]$, which of the following are true?

I. $f'(1.5) < f'(1.7)$

II. $10 < f'(1.5) < 30$

III. $f(1.7) > f''(1.7)$

(A) I only (B) II only (C) I and III only (D) II and III only (E) I, II, III

Ans

17. A particle moves on the xy-plane so that at time $t \geq 0$ its acceleration vector is $\langle 2, e^{-t} \rangle$. When $t = 0$, the particle is at rest and its position is $\langle 3,3 \rangle$. At $t = 2$ the position of the particle is

(A) $\langle 4, e^{-2} \rangle$ (B) $\langle 4, 2 + e^{-2} \rangle$ (C) $\langle 7, e^{-2} \rangle$ (D) $\langle 7, 2 + e^{-2} \rangle$ (E) $\langle 7, 4 + e^{-2} \rangle$

Ans

EXAM III
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Consider the following table of values for the differentiable function f.

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	5.0	3.5	2.6	2.0	1.5

(a) Estimate $f'(1.4)$.

(b) Give an equation for the tangent line to the graph of f at $x = 1.4$.

(c) What is the sign of $f''(1.4)$? Explain your answer.

(d) Using the data in the table, find a midpoint approximation with 2 equal subdivisions for

$$\int_{1.0}^{1.8} f(x)\,dx.$$

2. Consider the two curves defined by the equations $xy = \sqrt{2}$ and $x^2 - y^2 = 1$.

(a) Find the intersection points of the two curves.

(b) Show that the curves have perpendicular tangent lines at their points of intersection.

(c) Find the area of the first quadrant region below both curves from $x = 1$ to $x = 4$.

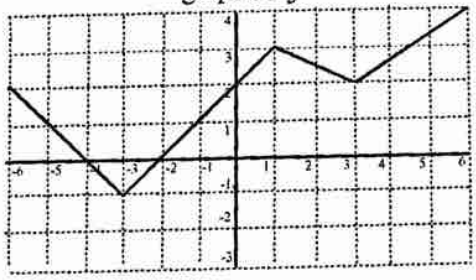
Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems

3. Let f be a function defined on the closed interval $-6 \le x \le 6$ with $f(1) = 5$. The graph of f', the derivative of f, consists of four line segments.

graph of f'



(a) Find $f(3)$.

(b) For $-6 \le x \le 6$, find all values of x at which the graph of f has a point of inflection. Justify.

(c) On what intervals is the graph of f both increasing and concave down?

(d) Let a second function g be defined by $g(x) = x^2 - 3x - 1$. Now, if a composite function H is defined by $H(x) = f[g(x)]$, find $H'(4)$.

4. a) Does the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n + 5}}$ converge? Justify your answer.

b) Determine all values of x for which the series $\sum_{n=1}^{\infty} \frac{3^n \cdot x^n}{n}$ converges. Justify your answer.

5. A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 6(2t+1)^{-3/2}$. When $t = 4$ the position of the particle is 9 and its velocity is 1.

(a) Write an equation for the velocity, $v(t)$, of the particle for all $t \geq 0$.

(b) Find the values of t for which the particle is at rest.

(c) Write an equation for the position, $s(t)$, of the particle for all $t \geq 0$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

6. Let f be a function whose domain is the closed interval [0, 5]. The graph of f is shown at the right.

Consider the function h defined by $h(x) = \int_0^{x^2} f(t)\, dt$.

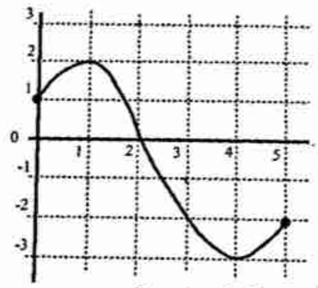
(a) Show that h is an even function.

(b) Find the domain of h.

(c) Find $h'(2)$.

(d) Give all values of x for which $h(x)$ is a maximum. Show the analysis that leads to your conclusion.

(e) Sketch a graph of $y = h(x)$ over its domain.



Graph of f

EXAM IV
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. The value of $\int_0^{\infty} e^{-x}dx$ is

(A) -1 (B) 0 (C) $\frac{1}{e}$ (D) 1 (E) nonexistent

Ans

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2. The graph of $f(x) = x\sin x$ defined on $0 < x < \pi$ has an inflection point whenever

(A) $\tan x = -\frac{2}{x}$

(B) $\tan x = \frac{2}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

(E) $\cos x = x$

Ans

3. The area of the region in the first quadrant bounded by the curve $y = \sqrt{2x+1}$ and the line $x = 4$ is equal to

(A) 2

(B) $\frac{16}{3}$

(C) $\frac{26}{3}$

(D) $\frac{35}{3}$

(E) $\frac{52}{9}$

Ans

4. $\lim_{x \to 3} \left[\frac{\ln\left(\frac{x-1}{2}\right)}{3-x} \right]$

(A) -1

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

(E) 1

Ans

5. The interval of convergence for the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ is

(A) $-\infty < x < \infty$

(B)) $-1 < x < 1$

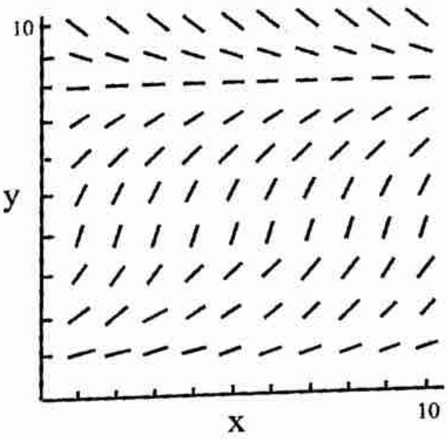
(C) $-1 \le x < 1$

(D) $-1 < x \le 1$

(E) $-1 \le x \le 1$

Ans

6. A slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure at the right. Which of the following statements are true?



I. The value of $\frac{dy}{dx}$ at the point (3, 3) is approximately 1.

II. As y approaches 8 the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of x.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

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7. $\lim_{h \to 0}\left(\frac{\tan(x+h) - \tan x}{h}\right) =$

(A) $\sec x$ (B) $-\sec x$ (C) $\sec^2 x$ (D) $-\sec^2 x$ (E) does not exist

Ans

---

8. $\int \frac{x^2 + 2x + 9}{x^2 + 9}\, dx =$

(A) $x + \frac{1}{8}\text{Arctan}\frac{x}{3} + C$

(B) $x + \frac{1}{4}\text{Arctan}\frac{x}{3} + C$

(C) $x + \frac{1}{2}\ln(x^2 + 9) + C$

(D) $1 + \ln(x^2 + 9) + C$

(E) $x + \ln(x^2 + 9) + C$

Ans

9. If $y = \ln(\cos x)$ and $0 < x < \frac{\pi}{2}$, what is $\frac{d^2y}{dx^2}$ in terms of x ?

(A) $\tan x$

(B) $-\tan x$

(C) $\sec^2 x$

(D) $-\sec^2 x$

(E) $-\csc^2 x$

Ans

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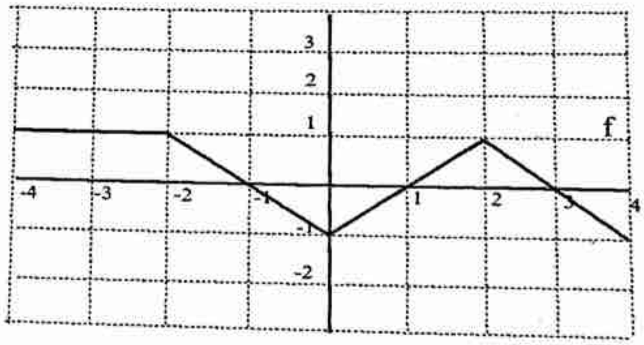
10. The graph of f is shown at the right. Which of the following statements must be true?



I. $f'(3) > f'(1)$

II. $\int_0^2 f(x)\,dx > f'(3.5)$

III. $\int_1^0 f(x)\,dx = \int_2^3 f(x)\,dx$

A) I only B) II only C) I and II only D) II and III only E) I, II and III

Ans

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11. The base of a solid is the region in the first quadrant bounded by the curve $y = \sqrt{\sin x}$ for $0 \le x \le \pi$. If each cross section of the solid perpendicular to the x-axis is a semi-circle, the volume of the solid is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{12}$

Ans

12. A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)(t-6)^3$. The particle is moving to the left for

(A) $t > 3$

(B) $2 < t < 6$

(C) $3 < t < 6$

(D) $0 \leq t < 3$

(E) $t > 6$

Ans

---

13. $\int x^3 \ln x \, dx =$

(A) $\frac{x^4}{4}(4\ln x - 1) + C$

B) $\frac{x^4}{16}(4\ln x - 1) + C$

(C) $\frac{x^2}{4}(\ln x - 1) + C$

(D) $3x^2(\ln x - \frac{1}{2}) + C$

(E) $x^2(3\ln x + 1) + C$

Ans

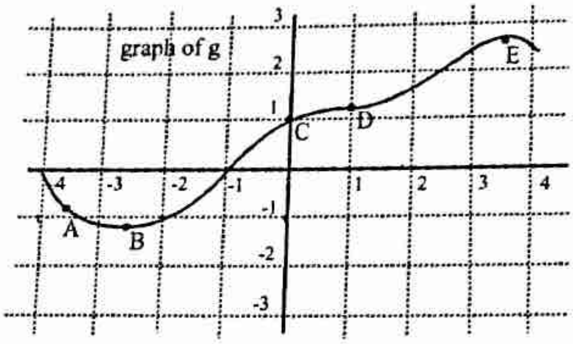
---

14. The slope of the tangent line to the curve $y(\cos x) + e^y = 5$ at the point where $x = \frac{\pi}{2}$ is

(A) 0 (B) 5 (C) $\frac{\ln 5}{5}$ (D) $\frac{(5 + \ln 5)}{5}$ (E) none of these

Ans

15. At which point on the graph of $y = g(x)$ below is $g'(x) = 0$ and $g''(x) < 0$?



(A) A (B) B (C) C (D) D (E) E

Ans

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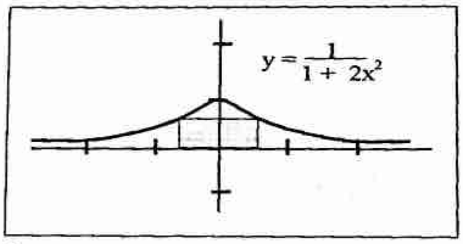
16. The height of the rectangle with the largest area that can be inscribed under the graph of $y = \dfrac{1}{1 + 2x^2}$ is



(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\sqrt{2}$

(E) none of these

Ans

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17. If $f(x) = \frac{x^2+1}{e^x}$, then the graph of f is decreasing and concave down on the interval

(A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, 3)$ (D) $(3, 4)$ (E) $(4, \infty)$

Ans

18. The function f is defined by $f(x) = x^3 + 1$. If f^{-1} is the inverse function of f and $h(x) = f^{-1}(x)$, then $h'(2)$ is

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) 1

(E) nonexistent

Ans

19. If $f(x) = \begin{cases} x^2 & \text{for } 0 \le x \le 2 \\ 6 - x & \text{for } x > 2 \end{cases}$, then $\int_0^4 f(x)\,dx$ is

(A) $21\frac{1}{3}$

(B) $18\frac{2}{3}$

(C) 16

(D) $8\frac{2}{3}$

(E) $4\frac{1}{3}$

Ans

20. An equation of the line normal to the graph of $f(x) = \frac{x}{x-2}$ at $(1, -1)$ is

(A) $2x - y - 3 = 0$

(B) $2x + y + 1 = 0$

(C) $x - 2y + 3 = 0$

(D) $x + 2y + 1 = 0$

(E) $x - 2y - 3 = 0$

Ans

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21. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1 + t,\ t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5, 0 \rangle$, then the position of the particle at $t = 2$ is

(A) $\langle 1, 12 \rangle$
(B) $\langle 4, 4 \rangle$
(C) $\langle 5, 9 \rangle$
(D) $\langle 9, 4 \rangle$
(E) $\langle 5, 0 \rangle$

Ans

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22. Let f be defined by $f(x) = x^{2/3}(2x - 5)$. f is decreasing on the interval

(A) $x < -\frac{5}{2}$ (B) $-\frac{5}{2} < x < 0$ (C) $x > 1$ (D) $0 < x < \frac{5}{8}$ (E) $0 < x < 1$

Ans

23. The average value of $f(x) = e^{2x} + 1$ on the interval $0 \le x \le \frac{1}{2}$ is

(A) e (B) $\frac{e}{2}$ (C) $\frac{e}{4}$ (D) $2e - 1$ (E) $\frac{e^{2x}+1}{2}$

Ans

24. If f is continuous at $x = 2$, and if $f(x) = \begin{cases} \dfrac{\sqrt{x+2} - \sqrt{2x}}{x-2} & \text{for } x \ne 2 \\ k \text{ for } x = 2 \end{cases}$

then $k =$

(A) $-\frac{1}{2}$
(B) $-\frac{1}{4}$
(C) 0
(D) $\frac{1}{4}$
(E) $\frac{1}{2}$

Ans

25. The approximate value of $y = \sqrt{x^2 + 3}$ at $x = 1.04$, obtained from the tangent to the graph at $x = 1$, is

(A) 2.01

(B) 2.02

(C) 2.03

(D) 2.04

(E) 2.05

Ans

26. Given the differential equation $\frac{dy}{dx} = x + y$ and $y(0) = 2$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) 3 (B) $\frac{7}{2}$ (C) $\frac{15}{4}$ (D) $\frac{19}{4}$ (E) $\frac{21}{4}$

Ans

27. Which of the following are true?

I. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, then $\sum_{n=1}^{\infty} b_n$ diverges.

II. The sum of the geometric series $\frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \ldots$ is 2.

III. If $\sum a_k$ diverges then $\lim_{k \to \infty} a_k \neq 0$.

(A) I only (B) II only (C) III only

(D) I and II only (E) II and III only

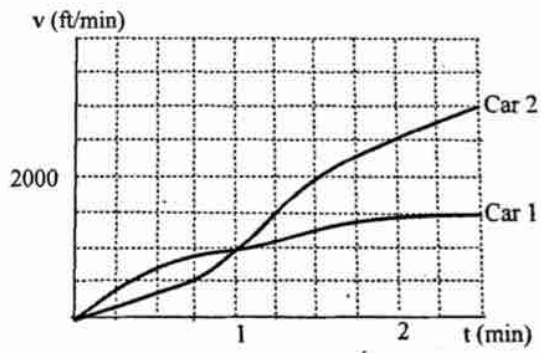
Ans

28. Two cars start from rest at a traffic light and accelerate for several minutes. The figure at the right shows their velocities as a function of time. Which of the following statements are true?

I. Car I is ahead at one minute.

II. Car II is ahead at two minutes.

III. Car I and Car II are accelerating at the same rate at $t = 1$.



(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, III

Ans

EXAM IV
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

1. Suppose a car is moving with increasing speed according to the following table.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

The closest approximation of the distanced traveled in the first 10 seconds is

(A) 150 ft (B) 250 ft (C) 350 ft (D) 450 ft (E) 550 ft

Ans

2. If $f(x) = \dfrac{\ln x^2 - x\ln x}{x-2}$ when $x \neq 2$, and f is continuous for all positive real numbers, then $f(2)$ is

(A) -1 (B) -2 (C) $-\frac{e}{4}$ (D) $-\ln 2$ (E) undefined

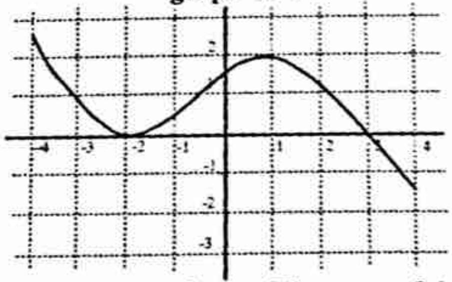
Ans

3. The graph of the **second derivative** of a function f is shown below. Which of the following are true about the original function f ?

I. The graph of f has an inflection point at $x = -2$.

II. The graph of f is concave down on the interval $(0, 4)$.

III. If $f'(0) = 0$, then f is increasing at $x = 2$.



(A) I only (B) II only (C) III only (D) I and II only (E) none of these

Ans

4. For $0 \le t \le 21$, the rate of change of the number of black flies on a coastal island at time t days is modeled by $R(t) = 3\sqrt{t}\cos\left(\frac{t}{3}\right)$ flys per day. There are 500 flies on the island at time $t = 0$. To the nearest whole number, what is the maximum number of flies for $0 \le t \le 21$.

(A) 500 (B) 510 (C) 520 (D) 530 (E) 540

Ans

5. A function f is defined for all real numbers and has the following property: $f(a+b) - f(a) = 3a^2b + 2b^2$. $f'(x)$ is

(A) 0

(B) 1

(C) $3x^2$

(D) $3x^2 + b$

(E) nonexistent

Ans

6. The position of a particle moving in the xy-plane is given by

$$x = t^2 + 2t, \quad y = 2t^2 - 6t.$$

What is the speed of the particle when $t = 2$?

(A) $2\sqrt{10}$

(B) $4\sqrt{10}$

(C) $6\sqrt{10}$

(D) $8\sqrt{10}$

(E) $10\sqrt{10}$

Ans

7. A point moves along the curve $y = x^2 + 1$ so that the x-coordinate is increasing at the constant rate of $\frac{3}{2}$ units per second. The rate, in units per second, at which the distance from the origin is changing when the point has coordinate (1, 2) is equal to

(A) 1.565 (B) 2.236 (C) 3.354 (D) 6.708 (E) 7.500

Ans

8. If $F(x) = \int_0^x \frac{\sin t}{1 + \cos t}\, dt$, then $F''\left(\frac{\pi}{3}\right)$ is

(A) $-\sqrt{3}$

(B) $-\frac{1}{2}$

(C) $4 - 2\sqrt{3}$

(D) $2 - \sqrt{3}$

(E) $\frac{2}{3}$

Ans

9. If the substitution $u = \sqrt{x+1}$ is made in $\int_0^3 \frac{1}{x\sqrt{x+1}}\,dx$, the resulting integral is

(A) $\int_1^2 \frac{1}{u^2-1}\,du$ (B) $\int_1^2 \frac{2}{u^2-1}\,du$ (C) $\int_0^3 \frac{1}{(u-1)(u+1)}\,du$

(D) $2\int_1^2 \frac{1}{u(u^2-1)}\,du$ (E) $2\int_0^3 \frac{1}{u^2-\mathrm{u}}\,du$

Ans

---

10. The functions f and g are defined on the closed interval $[0, \mathrm{b}]$ by $f(x) = \cos(2x)$ and $\mathrm{g}(x) = \mathrm{e}^x - 1$. They will have the same average value if b is

(A) 0.848 (B) 0.852 (C) 0.854 (D) 0.858 (E) 0.862

Ans

11. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)3^n}$ is

(A) ∞ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6

Ans

12. Which of the following functions has a derivative at $x = 0$?

I. $y = \arcsin(x^2 - 1) - x$

II. $y = x \cdot |x|$

III. $y = \sqrt{x^4}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

13. Let $F(x)$ be an antiderivative of $f(x) = \frac{4\ln x}{e^{\sqrt{x}}}$. If $F(1) = 2$, then $F(3)=$

A) 2.837 (B) 3.007 (C) 3.177 (D) 3.347 (E) 3.517

Ans

14. The following table lists the known values of a function f.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

If the Trapezoid Rule is used to approximate $\int_1^5 f(x)\,dx$ the result is

(A) 4.1

(B) 4.3

(C) 4.5

(D) 4.7

(E) 4.9

Ans

15. Which of the following statements are true about the function f if its derivative f' is defined by $f'(x) = x(x-a)^3,\ a > 0.$

I. The graph of f is increasing at $x = 2a$.

II. The function f has a local maximum at $x = 0$.

III. The graph of f has an inflection point at $x = a$.

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, III

Ans

16. The approximate total area of the region enclosed by the polar graph of $r = \sin(2\theta)$ is

(A) 0.393
(B) 0.785
(C) 1.178
(D) 1.571
(E) 1.873

Ans

17. The average rate of change of the differentiable function f from $(3, f(3))$ to $(x, f(x))$ is given by $\dfrac{x^2 - x - 6}{x - 3}$. The value of $f'(3)$ is

(A) 0
(B) 1
(C) 3
(D) 5
(E) undefined

Ans

EXAM IV
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let f be the function defined by $f(x) = e^{x/2} - \ln(x^3 + 1)$ for $x > -1$.

(a) Find the x-coordinate of all relative maximum and minimum points. Justify your answers.

(b) Find the intervals on which f is increasing.

(c) Find the intervals on which the graph of f is concave down.

(d) Find the area of the first quadrant region bounded by the graph of f and the lines $x = 0$ and $x = 1$.

2. Consider the curve C given by the parametric equations

$$x = 2 - 3\cos t \quad \text{and} \quad y = 3 + 2\sin t\ , \quad \text{for } -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

(b) Find an equation of the tangent line at the point where $t = \frac{\pi}{4}$.

(c) The curve C intersects the y-axis twice. Find the length of the curve between the two y-intercepts.

Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems in part A without the use of a calculator.

3. Car A has positive velocity $v(t)$ as it travels along a straight road, where v is a differentiable function of t. The velocity of the car is recorded for several selected values of t over the interval $0 \le t \le 60$ seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60
$v(t)$ (feet per second)	5	14	7	11	12	40	44

(a) Use the data from the table to approximate the acceleration of Car A at $t = 25$ seconds. Show the computation that lead to your answer. Indicate units of measure.

(b) Use the data from the table to approximate the distance traveled by Car A over the time interval $0 \le t \le 60$ seconds by using a midpoint Riemann sum with 3 subdivisions of equal length. Show the work that lead to your answer.

(c) Car B travels along the same road with an acceleration of $a(t) = \frac{1}{\sqrt{t+9}}$ ft / sec^2.

At time $t = 0$ seconds, the velocity of Car B is 3 ft/sec. Which car is traveling faster at $t = 40$ seconds? Show the work that lead to your answer

4. Let f be the function defined by the power series $f(x) = \sum_{n=0}^{\infty} a_n x^{2n}$ where $a_0 = 1$ and $a_n = \frac{a_{n-1}}{n}$ for $n \geq 1$.

(a) Write the first 5 terms of the series and the general term.

(b) Determine the radius of convergence for the series in part (a). Show your reasoning.

(c) Show that $f'(x) = 2x \cdot f(x)$.

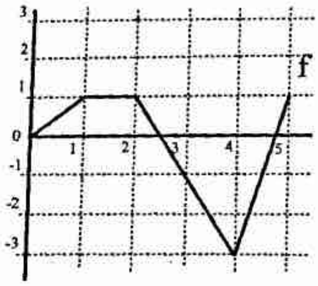
5. When the valve at the bottom of a cylindrical tank is opened, the rate at which the level of liquid in the tank drops is proportional to the square root of the depth of the liquid. Thus, if $y(t)$ is the liquid's depth at time t minutes after the valve is opened, water drains from the tank according to the differential equation $\frac{dy}{dt} = -k\sqrt{y}$ for some positive constant k that depends on the size of the drain.

(a) Find a general solution for the differential equation.

(b) Suppose that $y(0) = 9$ feet and $y(20) = 4$. Find an equation for $y(t)$.

(c) At what time is the water level dropping at a rate of 0.1 feet per minute?

6. Let f be a function defined the closed interval $[0, 5]$ with zeros at $x = 0$, $5/2$ and $19/4$. The graph of f is shown at the right.



Consider the function G defined by $G(x) = \int_0^x f(t)\,dt$.

(a) Find $G(3)$.

(b) On what intervals is the graph of G increasing and concave down? Show your reasoning.

(c) Show that G has exactly one zero between $x = 3$ and $x = 4$.

(d) Find an equation of the tangent line to the graph G at the point where $x = 3$.

(e) Sketch a graph of the function G over its domain.

EXAM V
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. $\int_2^3 \frac{x}{x-1}\,dx =$

(A) $1-\ln 2$
(B) $\ln 2$
(C) $1+\ln 2$
(D) $2+\ln 2$
(E) $5+\ln 2$

Ans ☐

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2. The radius of convergence of the series $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$ is 1. If the function f is defined by $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^2}$, then the interval of convergence for the series representing the derivative $f'(x)$ is

(A) $-\infty < x < \infty$
(B) $-1 < x < 1$
(C) $-1 \le x < 1$
(D) $-1 < x \le 1$
(E) $-1 \le x \le 1$

Ans ☐

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3. The position vector of a particle moving in the xy-plane at time t is given by $\mathbf{p} = (3t^2 - 4t)\,\mathbf{i} + (t^2 + 2t)\,\mathbf{j}$. The speed of the particle at $t = 2$ is

(A) 2

(B) $2\sqrt{10}$

(C) 10

(D) 14

(E) 20

Ans

4. If $f(x) = \ln(x^2 - e^{2x})$, then $f'(1) =$

(A) 0

(B) 1

(C) 2

(D) e

(E) undefined

Ans

5. The length of the curve $y = \int_0^x \sqrt{\frac{u}{3}}\, du$ from $x = 0$ to $x = 9$ is

(A) 16

(B) 14

(C) $10\frac{1}{3}$

(D) $9\sqrt{3}$

(E) $4\frac{2}{3}$

Ans

6. If a population of wolves grows according to the logistic equation

$$\frac{dN}{dt} = 0.05N - 0.0005N^2$$

where N is the number of wolves and t is measured in years, then $\lim_{t\to\infty} N(t) =$

(A) 50 (B) 75 (C) 100 (D) 150 (E) 200

Ans

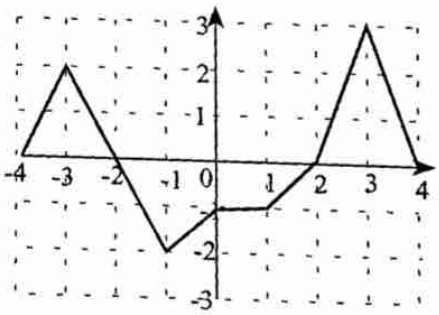
7. A particle moves along a straight line with its position at any time $t \geq 0$ given by

$s(t) = \int_0^t (x^3 - 2x^2 + x)\ dx$, where s is measured in meters and t in seconds. The maximum velocity attained by the particle on the interval $0 \leq t \leq 3$ is

(A) 1 m/s (B) 4 m/s (C) 9 m/s (D) 12 m/s (E) 16m/s

Ans

8. The graph of the function f on the interval $[-4,4]$ is shown at the right.



$$\int_{-4}^{4} |f(x)|\ dx =$$

(A) 1

(B) 2

(C) 5

(D) 8

(E) 9

Ans

9. Both x and y are functions of a third variable t and $y^2 + x^2 + y = 10$. If $\frac{dx}{dt} = -5$ when $x = 2$ and $y = 2$, then $\frac{dy}{dt} =$

(A) -1
(B) 1
(C) 2
(D) 3
(E) 4

Ans

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10. The substitution $u = \ln x$ transforms the definite integral $\int_1^e \frac{1 - \ln x}{x^2}\, dx$ into

(A) $\int_0^1 (1-u)\, du$
(B) $\int_0^e (1-u)\, du$
(C) $\int_0^1 \frac{1-u}{e^u}\, du$
(D) $\int_0^1 \frac{1-u}{e^{2u}}\, du$
(E) $\int_0^e \frac{1-u}{e^u}\, du$

Ans

---

11. The number of cells of a certain type of bacteria increases continuously at a rate equal to two more than three times the number of bacteria present. If there are 10 present at the start and 42 present t hours later, the value of t is

(A) $3 \ln 4$
(B) $\ln 4$
(C) $\frac{1}{2} \ln 4$
(D) $\frac{1}{3} \ln 4$
(E) $\frac{1}{4} \ln 4$

Ans

12. If $\frac{dy}{dx} = x \cdot \sec y$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $y = 0$ when $x = \sqrt{2}$, then when $x = 1$ the value of y is

(A) $-\frac{\pi}{6}$

(B) 0

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

(E) $\frac{\pi}{2}$

Ans

13. What is the approximation of the value of e^4 using the fourth degree Taylor polynomial about $x = 0$ for $e^{\sqrt{x}}$?

(A) $\frac{1}{3}$

(B) $\frac{4}{3}$

(C) 7

(D) 11

(E) 31

Ans

14. If $F(x) = 3x \sin x - \cos x - \frac{3\pi}{2}$, then an equation of the line tangent to the graph of F at the point where $x = \frac{\pi}{2}$ is

(A) $y = 3x - \frac{\pi}{2}$

(B) $y = 3(x - \frac{\pi}{2})$

(C) $y = 4x$

(D) $y = 4x - \frac{\pi}{2}$

(E) $y = 4x - 2\pi$

Ans

15. The function $f(x) = \begin{cases} 4 - x^2 & \text{for } x \le 1 \\ mx + b & \text{for } x > 1 \end{cases}$ is continuous and differentiable for all real numbers. The values of m and b are

(A) $m = 2, b = 1$

(B) $m = 2, b = 5$

(C) $m = -2, b = 1$

(D) $m = -2, b = 5$

(E) none of these

Ans ☐

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16. $\int \frac{8}{(x-1)(x+3)}\, dx =$

(A) $2 \ln\left|\frac{x+3}{x-1}\right| + C$

(B) $2 \ln\left|\frac{x-1}{x+3}\right| + C$

(C) $2 \ln|(x+3)(x-1)| + C$

(D) $2 \ln\left|\frac{1}{(x+3)(x-1)}\right| + C$

(E) $8 \ln\left|\frac{1}{(x+3)(x-1)}\right| + C$

Ans ☐

---

17. If $f(x) = \frac{x-k}{x+k}$ and $k \ne 0$, then $f''(0) =$

(A) $-\frac{4}{k^2}$ (B) $-\frac{2}{k}$ (C) 0 (D) $\frac{2}{k}$ (E) $\frac{4}{k^2}$

Ans ☐

18. The base of a solid is a right triangle whose perpendicular sides have lengths 6 and 4. Each plane section of the solid perpendicular to the side of length 6 is a semicircle whose diameter lies in the plane of the triangle. The volume of the solid is

(A) 2π

(B) 4π

(C) 8π

(D) 16π

(E) 24π

Ans

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19. $\lim_{h \to 0} \frac{(1+h)^3 - 1}{h} =$

(A) undefined

(B) 3

(C) 2

(D) 1

(E) 0

Ans

20. Suppose a function f is defined so that it has derivatives $f'(x) = x^2(1-x)$ and $f''(x) = x(2-3x)$. Over which interval is the graph of f both increasing and concave up?

(A) $x < 0$ (B) $0 < x < \frac{2}{3}$ (C) $\frac{2}{3} < x < 1$ (D) $x > 1$ (E) none of these

Ans

21. The average value of the function $f(x) = \sqrt[3]{x^2}$ on the interval $[0,8]$ is

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{4}$ (D) $\frac{12}{5}$ (E) $\frac{17}{6}$

Ans

22. Let $f(x) = \begin{cases} 2 & \text{if } x < 0 \\ x+2 & \text{if } x \geq 0 \end{cases}$ and let $F(x) = \int_{-2}^{x} f(t)\,dt$. Which of the following statements are true?

I. $F(1) = 6.5$ II. $F'(1) = 3$ III. $F''(1) = 1$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

23. Which of the following three improper integrals converge?

I. $\int_1^\infty \frac{1}{x^3}\,dx$ II. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ III. $\int_0^1 \frac{1}{x^3}\,dx$

(A) II only (B) I and II only (C) I and III only (D) II and III only (E) I, II, III

Ans

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24. The acceleration of a particle moving along the x-axis at time $t > 0$ is given by $a(t) = \frac{1}{t^2}$.

When $t = 1$ second, the particle is at $x = 2$ and moving with velocity -1 unit per second. The position when $t = \text{e}$ seconds is

(A) $x = -2$

(B) $x = -1$

(C) $x = 0$

(D) $x = 1$

(E) $x = 2$

Ans

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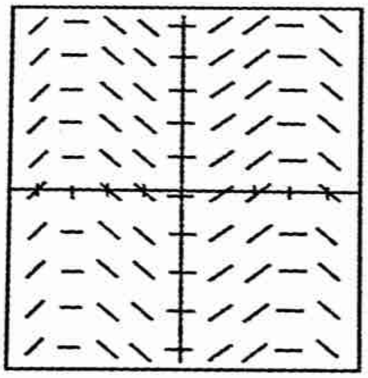
25. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = \tan x \cdot \sec x$

(B) $\frac{dy}{dx} = \sin x$

(C) $\frac{dy}{dx} = \sec^2 x$

(D) $\frac{dy}{dx} = \ln x$

(E) $\frac{dy}{dx} = e^{2x}$

Ans

26. The area enclosed by the two curves $y = x^2 - 4$ and $y = x - 4$ is given by

(A) $\int_0^1 (x - x^2)\, dx$ (B) $\int_0^1 (x^2 - x)\, dx$

(C) $\int_0^2 (x - x^2)\, dx$ (D) $\int_0^2 (x^2 - x)\, dx$

(E) $\int_0^4 (x^2 - x)\, dx$

Ans

27. The coefficient of x^3 in the Taylor series for e^{2x} at $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{8}{3}$

Ans

28.

x	1	3	6	8
$f(x)$	3	k	9	11

A function f is continuous on the closed interval [1, 8] and has values given in the table above. The trapezoidal approximation for $\int_1^8 f(x)\, dx$ found with 3 subintervals indicated by the data in the table is 49. What is the value of k?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Ans

EXAM V
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. The graph of $g(x) = \int_0^x \sin(2t)\,dt$ is both decreasing and concave up everywhere on the interval

(A) $(0, \frac{\pi}{4})$ (B) $(\frac{\pi}{4}, \frac{\pi}{2})$ (C) $(\frac{\pi}{2}, \frac{3\pi}{4})$ (D) $(\frac{3\pi}{4}, \pi)$ (E) none of these

Ans

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2. The position at time $t > 0$ of a particle moving on the x-axis is given by $x(t) = \sin(t)\cdot\cos(t)$ At the first instant when the acceleration is 1 unit per sec^2, the particle has velocity

(A) – 1 unit per sec
(B) –0.866 units per sec
(C) 0 units per sec
(D) 0.866 units per sec
(E) 1 unit per sec

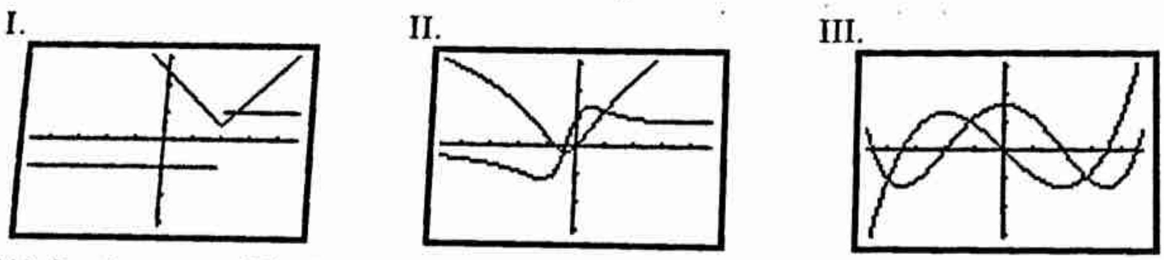
Ans

3. The average value of $f(x) = x \cdot \ln x$ on the interval $1 \le x \le e$ is

(A) 0.772 (B) 1.221 (C) 1.359 (D) 1.790 (E) 2.097

Ans

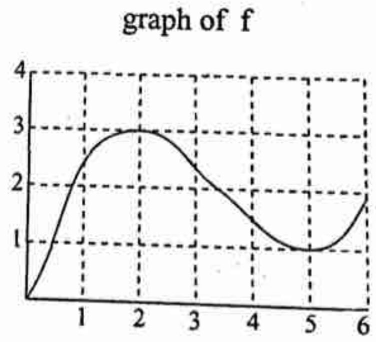
4. A pair of functions is graphed in each of the following viewing rectangles. In which of these is one function the derivative of the other?



(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

5. A graph of the function f is shown at the right. If the graph of f has horizontal tangents at $x = 2$ and $x = 5$, which of the following statements are true?



I. $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h} = f'(5)$

II. $\dfrac{f(5) - f(2)}{5 - 2} = \dfrac{2}{3}$

III. $f''(1) \le f''(5)$

(A) I and II only (B) I and III only (C) II and III only (D) I, II, III (E) none

Ans

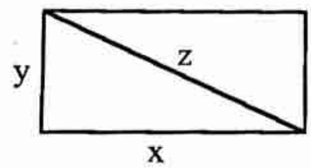
6. Let the function f be differentiable on the interval $[0, 2.5]$ and define g by $g(x) = f[f(x)]$. Use the table to estimate $g'(1)$.

x	0.0	0.5	1.0	1.5	2.0	2.5
$f(x)$	1.7	1.8	2.0	2.4	3.1	4.4

(A) 0.8 (B) 1.2 (C) 1.6 (D) 2.0 (E) 2.4

Ans

7. The diagonal z of the rectangle at the right is increasing at the rate of 2 cm/sec and $\frac{dy}{dt} = 3\frac{dx}{dt}$. At what rate is the length x increasing when $x = 3$ cm and $y = 4$ cm ?



(A) 1 cm/sec (B) $\frac{3}{4}$ cm/sec (C) $\frac{2}{3}$ cm/sec (D) $\frac{1}{3}$ cm/sec (E) $\frac{1}{15}$ cm/sec

Ans

8. The position of a particle moving on the x-axis, starting at time $t = 0$, is given by $x(t) = (t - a)^3(t - b)$ where $0 < a < b$. Which of the following statements are true?

I. The particle is at a positive position on the x-axis at time $t = \frac{a+b}{2}$.

II. The particle is at rest at time $t = a$.

III. The particle is moving to the right at time $t = b$.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

Ans

9. Let $f(x) = \dfrac{\ln e^{x+1}}{2x}$ for $x > 0$. If g is the inverse of f, then $g'(1) =$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

10. Let $F(x) = \cos(2x) + e^{-x}$. For what value of x on the interval $[0,3]$ will F have the same instantaneous rate of change as the average rate of change of F over the interval ?

(A) 1.542 (B) 1.610 (C) 1.678 (D) 1.746 (E) 1.814

Ans

11. Which of the following series converge?

(A) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (B) $\sum_{n=1}^{\infty} \frac{1}{n}$ (C) $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ (D) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ (E) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

Ans

12. If f is the antiderivative of $\frac{1}{\sqrt{1+x^3}}$ such that $f(0)=2$, then $f(4)=$

(A) 3.205 (B) 3.355 (C) 3.505 (D) 3.655 (E) 3.805

Ans

13. The rate at which water is being pumped into a tank is $r(t)=20e^{0.02t}$, where $t \geq 0$ is in minutes and $r(t)$ in gallons per minute. Approximtely how many gallons of water are pumped into the tank during the first five minutes?

(A) 20 (B) 22 (C) 85 (D) 105 (E) 150

Ans

14. The curve passing through (1, 0) satisfies the differential equation $\frac{dy}{dx} = 4x + y$. An approximation to $y(2)$ using Euler's Method with two equal steps is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Ans

15. Let f be a function having derivatives of all orders for all real numbers, and $\left|f^{(4)}(x)\right| \le 3$ for all x in the interval [0, 2]. If the third-degree Taylor polynomial for f about $x = 0$ is used to approximate f on the interval [0, 2], what is the Lagrange error bound for the maximum error on the interval [0, 2]?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 18

Ans

16. The area of the region bounded by the graphs of $y = \ln(x+4)$ and $y = 0.5x^2$ is

(A) 1.884 (B) 3.089 (C) 4.294 (D) 5.449 (E) 6.704

Ans

17. If f is a continuous function that is defined for all real numbers with

$\int_1^3 f(x)\,dx = \frac{5}{2}$ and $\int_1^5 f(x)\,dx = 10$, then $\int_3^5 [2f(x)+6]\,dx =$

(A) 18 (B) 21 (C) 24 (D) 27 (E) 30

Ans

EXAM V
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.
- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.
- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

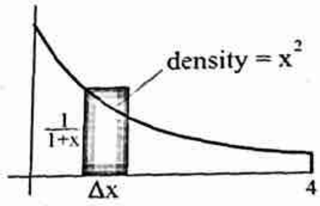
1. A line is drawn tangent to the curve with equation $y = e^x$ at the point P with coordinates (a, e^a).

(a) Determine an equation of the tangent line at the point (a, e^a) and find the coordinates of the point R at which the tangent line crosses the x-axis.

(b) Consider the triangle ΔPOR, where O is the origin. Express the area of ΔPOR as a function of a.

(c) Find the value of a, with $-1 \le a < 1$ for which the area of ΔPOR is a maximum. Justify your answer.

2. A flat metal plate one centimeter thick is shaped like the first quadrant region under the graph of $f(x) = \dfrac{1}{1+x}$ between $x = 0$ and $x = 4$. The density of the plate varies; at a distance x units from the y-axis the density is given by x^2 grams/cm^3.



(a) Assume the approximate mass, ΔM, of a strip of width Δx is:

$$\Delta M = (\text{volume})(\text{density}).$$

Write a left-hand Riemann sum with 4 terms that approximates the total mass of the metal plate.

(b) Write a definite integral that gives the exact value of the total mass of the plate.

(c) Determine the total mass of the plate.

Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

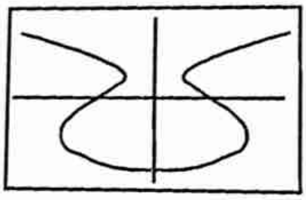
- During the timed portion for part B, you may go back and continue to work on the problems in part A without the use of a calculator.

3. Let f be a function with derivatives of all orders and for which $f(2) = 0$. The n^{th} derivative of f at $x = 2$ is given by $f^{n}(2) = \dfrac{n!}{n \cdot 3^{n}}$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$.

(b) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.

4. A graph of $y^3 + y^2 - 5y - x^2 = -4$ is shown at the right.



(a) Find $\frac{dy}{dx}$ in terms of x and y.

(b) Write an equation for the line tangent to the curve at the point (2, 0).

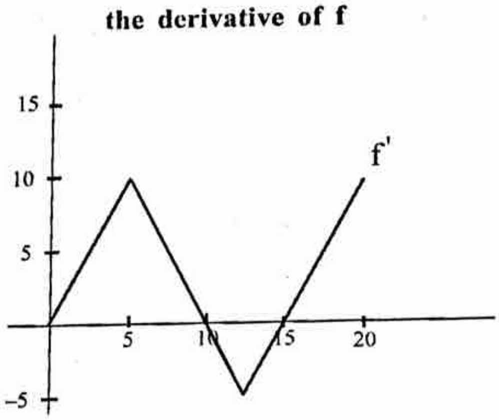
(c) Find the x-coordinates of the points at which the graph of the equation has vertical tangent lines or horizontal tangent lines. Justify your answer.

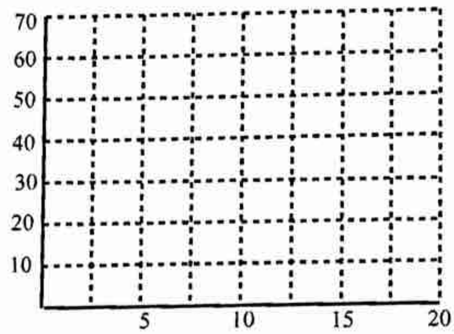
5. The figure shows f', the derivative of a function f. The domain of the function f is the interval [0, 20].

(a) For what values of x, $0 < x < 20$, does f have a relative maximum? Justify your answer.

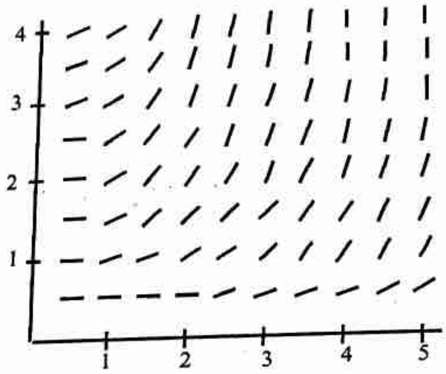
(b) For what values of x is the graph of f concave down? Justify your answer.

(c) If $f(0) = 10$, sketch a graph of the function f on the axes provided. List the coordinates of all critical points and inflection points.





6. The slope field for the differential equation $\frac{dy}{dx} = 0.5xy$, $0 \le x \le 5$ and $0 \le y \le 4$, is shown in the figure at the right.



(a) Sketch the solution curve that satisfies the initial condition $y(0) = 2$ on the slope field.

(b) Solve the differential equation and find the particular solution that contains the point $(0, 2)$.

(c) Use the function in part (b) to calculate the exact value of y when $x = 2$.

(d) Starting at the point $(0, 2)$, use Euler's method with 2 steps of size 1 to estimate $y(2)$. How does this value compare with the exact value in part (c)?

EXAM VI
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. Which of the following is an antiderivative of $\sqrt{4-2x}$?

(A) $-\frac{1}{3}(4-2x)^{3/2}$ (B) $\frac{2}{3}(4-2x)^{3/2}$ (C) $-\frac{1}{6}(4-2x)^{3}$

(D) $\frac{1}{2}(4-2x)^{2}$ (E) $\frac{4}{3}(4-2x)^{3/2}$

Ans ☐

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2. A particle moves along a straight line with its position at any time $t \geq 0$ given by $s(t) = \int_0^t (\sqrt{x} - x + 1)\,dx$, where s is measured in meters and t in seconds. What is the velocity of the particle when its acceleration is zero?

(A) $-\frac{1}{4}$ m/s (B) $\frac{1}{4}$ m/s (C) $\frac{1}{2}$ m/s (D) 1 m/s (E) $\frac{5}{4}$ m/s

Ans ☐

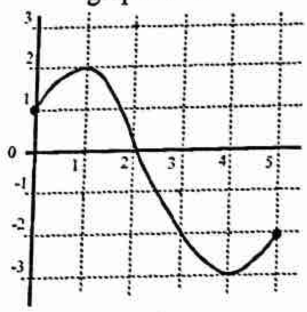
3. If $\frac{dy}{dx} = \frac{\cos x}{e^y}$ and $y(0) = 0$, then $y\left(\frac{\pi}{2}\right) =$

(A) 0
(B) ln 2
(C) 1
(D) $\frac{1}{2}$
(E) −1

Ans

---

4. The function h is defined on the interval $0 \le x \le 5$ and a graph of its derivative function h' is shown in the figure. Which of the following are true?



I. The function h is decreasing on the interval (1, 2).

II. The function h has a local maximum at the point where $x = 2$.

III. Given $h(1) = -1$, an equation of the tangent line to the graph of h at the point $(1, -1)$ is $y = 2x - 3$.

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

---

5. $\int_1^{\infty} \frac{x}{1+x^2}\,dx$ is

(A) 0 (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{\pi}{2}$ (E) divergent

Ans

6. If $x = \sin t$ and $y = \cos^2 t$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $-\frac{1}{4}$ (D) -2 (E) 2

Ans

7. In an effort to enhance fishing, 100 trout were initially put in a small lake. Fishery Department biologists predict that the rate of growth of the trout population is modeled by the logistic differential equation $\frac{dP}{dt} = 0.12P\left(1 - \frac{P}{600}\right)$, where time t is measured in months.

I. The growth rate of the fish population is greatest at $P = 600$.

II. If $P > 600$, the population of fish is decreasing.

III. $\lim_{t \to \infty} P(t) = 600$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Ans

8. If f and g are both continuous and differentiable functions for all real numbers, which of the following definite integrals is equal to $f(g(5)) - f(g(3))$?

(A) $\int_3^5 f(g'(x)) \cdot g(x)\, dx$

(B) $\int_3^5 f'(g(x)) \cdot g(x)\, dx$

(C) $\int_3^5 f'(g(x)) \cdot g'(x)\, dx$

(D) $\int_3^5 f(g(x)) \cdot f'(x)\, dx$

(E) $\int_3^5 f'(g(x))\, dx$

Ans

9. $\lim\limits_{x \to +\infty} \dfrac{x - \frac{1}{2x}}{2x - \frac{1}{6x}} =$

(A) -3 (B) $-\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 2

Ans

0. An equation of the line tangent to the graph of $y = \dfrac{3x+4}{4x-3}$ at the point where $x = 1$ is

(A) $y + 25x = 32$ (B) $y - 31x = -24$ (C) $y - 7x = 0$
(D) $y + 5x = 12$ (E) $y - 25x = -18$

Ans

11. If $\dfrac{dy}{dx} = \sqrt{x}$, then the average rate of change of y with respect to x on the closed interval $[0, 4]$ is

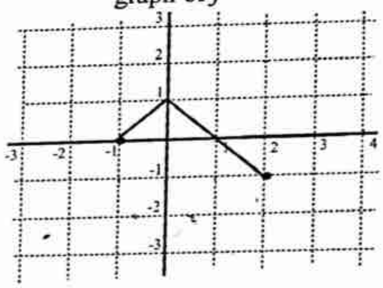
(A) $\frac{1}{16}$ (B) 1 (C) $\frac{4}{3}$ (D) $\sqrt{2}$ (E) 2

Ans

---

12. A graph of the function f, consisting of two line segments, is shown in the figure. If $g(x) = \int_1^x f(t)\,dt$, then the maximum value of g on the closed interval $[-1,2]$ is

graph of f



(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) 1 (E) 2

Ans

---

13. The total area of the region enclosed by the polar graph of $r = 1 + \cos\theta$ is given by which of the following expressions?

(A) $\frac{1}{2}\int_0^{\pi}(1+\cos\theta)^2\,d\theta$

(B) $\int_0^{\pi}(1+\cos\theta)^2\,d\theta$

(C) $\frac{1}{2}\int_0^{2\pi}(1+\cos\theta)\,d\theta$

(D) $2\int_0^{2\pi}(1+\cos\theta)^2\,d\theta$

(E) $\int_0^{2\pi}(1+\cos\theta)^2\,d\theta$

Ans

14. A particle moves along the x-axis with acceleration given by $a(t) = \cos t$ ft/sec² for $t \geq 0$. At time $t = 0$ seconds the velocity of the particle is 2 ft/sec. The total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$ is

(A) 1 ft (B) $\frac{\pi}{2}$ ft (C) π ft (D) $\pi+1$ ft (E) $\pi+2$ ft

Ans

15. A curve is parametrically defined by the equations $x = 2\cos t$ and $y = 2\sin t$. The length of the arc from $t = 0$ to $t = 2$ is

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

Ans

16. $\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^{2k} =$

(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{9}{8}$ (E) ∞

Ans

17. The shortest distance from the curve $xy = 4$ to the origin is

(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $2\sqrt{2}$ (E) $\frac{1}{2}\sqrt{2}$

Ans

18. If the function f is defined so that $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = 0$, which of the following must be true?

(A) f is not continuous at $x = 0$

(B) $f(a) = 0$

(C) $f'(a) = 0$

(D) $f(a)$ does not exist

(E) $\lim_{x \to a} f(a)$ does not exist

Ans

19. A particle moves in the xy-plane so that its velocity vector at time $t \geq 0$ is $v(t) = (2t, \sin t)$ and the particle's position vector at time $t = 0$ is (0, 1). The position vector of the particle when $t = \pi$ is

(A) $(\pi^2 + 1, 3)$ (B) $(\pi^2, 3)$ (C) $(\pi^2, 2)$ (D) $(-\pi^2, 3)$ (E) $(-\pi^2, 2)$

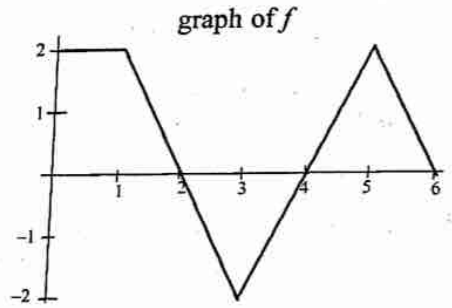
Ans

20. The density of cars (in cars per mile) along a 20-mile stretch of the Jersey Turnpike starting at a toll plaza is given by $f(x) = 500 + 100\sin(\pi x)$ where x is the distance in miles from the toll plaza and $0 \le x \le 20$. The total number of cars along the 20-mile stretch is

(A) 8500 (B) 9000 (C) 9500 (D) 10,000 (E) 10,500

Ans

21. The function G is defined on the interval [0, 6] by $G(x) = \int_0^x f(t)\,dt$ where f is the function graphed in the figure. A linear approximation of G near $x = 3$ is



(A) $6 - x$ (B) $8 - x$ (C) $5 - 2x$ (D) $8 - 2x$ (E) $9 - 2x$

Ans

22. What is the radius of convergence for the series $\sum_{n=0}^{\infty} \frac{3^n (x+2)^n}{n+1}$?

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) 2

Ans

23. The infinite first quadrant region bounded above by the curve $y = e^{-2x}$ is rotated about the x-axis to generate a solid of revolution. The volume of the solid is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) ∞

Ans

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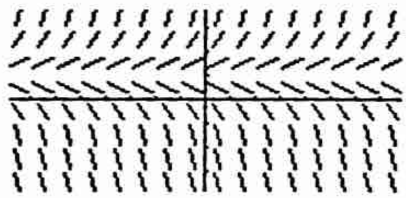
24. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. Which of the following statements are true?



I. A solution curve that contains the point (0, 2) also contains the point (–2, 0).

II. As y approaches 1 the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of y.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

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25. If the definite integral $\int_1^7 \ln x \, dx$ is approximated by 3 circumscribed rectangles of equal width on the x-axis, then the approximation is

(A) $\frac{1}{2}(\ln 3 + \ln 5 + \ln 7)$

(B) $\frac{1}{2}(\ln 1 + \ln 3 + \ln 5)$

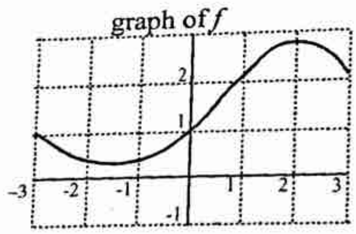
(C) $2(\ln 3 + \ln 5 + \ln 7)$

(D) $2(\ln 3 + \ln 5)$

(E) $\ln 1 + 2\ln 3 + 2\ln 5 + \ln 7$

Ans

26. The function f is defined on the interval $-3 \le x \le 3$ and its graph is shown in the figure. If the graph of f has a horizontal tangent at $x = 2$, which of the following statements are true?



I. $f(2) > f'(2)$

II. $\int_0^1 f(x)\,dx > f''(2)$

III. $1 - x + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{15} - \cdots$ is a Maclaurin series representation of the function f

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II and III

Ans

27. Given the differential equation $\frac{dy}{dx} = \frac{10x}{x+y}$ and $y(0) = 2$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) $\frac{16}{3}$

Ans

28. The coefficient of $(x-1)^5$ in the Taylor series for $x \ln x$ about $x = 1$ is

(A) $-\frac{1}{20}$ (B) $\frac{1}{20}$ (C) $-\frac{1}{24}$ (D) $\frac{1}{24}$ (E) $-\frac{1}{120}$

Ans

EXAM VI
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t+1)(t-4)^3$. For what value of t, $2 \leq t \leq 4$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [2, 4]?

(A) 2.544 (B) 2.644 (C) 2.744 (D) 2.844 (E) 2.944

Ans

---

2. If the function f defined by $f(x) = x^4 + ax^2 + 8x - 5$ has a horizontal tangent line and a point of inflection at the same value of x, then a is

(A) −12 (B) −6 (C) 0 (D) 1 (E) 3

Ans

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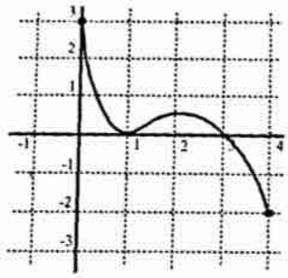
3. A graph of the second derivative of a function f is shown in the figure. Use the graph to determine which of the following are true.

I. The f-graph is concave down on the interval (1, 3).

II. The f-graph has points of inflection at $x = 1$ and $x = 3$.

III. If $f'(2) = 0$, f is increasing at $x = 3$.

graph of f''



(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

4. The area of the first quadrant region bounded above by the graph of $y = 2\sin x$ and below by the graph of $y = e^{x/2}$ is

(A) 2.312

(B) 1.398

(C) 1.343

(D) 0.429

(E) none of these

Ans

5. The base of a solid is the region bounded below by the curve $y = x^2$ and above by the line $y = d$, where d is a positive constant. Every cross-section of the solid perpendicular to the y-axis is a square. If the volume of the solid is 72, what is the value of d?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

Ans

6. The function g is defined by $g(x) = \int_{\pi/2}^{x} \cos t \, dt$. The maximum value of g on the closed interval $[-\pi, \pi]$ is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

7. The Cartesian equation for the polar curve $r = \sin\theta$ is

(A) $x^2 + y^2 = x$

(B) $x^2 + y^2 = y$

(C) $x^2 + y^2 = x + y$

(D) $(x + y)^2 = y$

(E) $(x + y)^2 = x$

Ans

8. The function f is defined for all real numbers by $f(x) = \begin{cases} e^{-x} + 3, & \text{for } x > 0, \\ ax + b & \text{for } x \le 0. \end{cases}$ If f is differentiable at $x = 0$, then $a + b =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Ans

9. If the derivative of the function f is given by $f'(x) = \cos\left(\frac{x}{2}\right) \cdot \ln x$ for $0 < x < 3\pi$, then the graph of f is increasing and concave down on the interval

(A) (0, 1.967) (B) (1.967, 3.141) (C) (3.141, 6.601)
(D) (6.601, 9.424) (E) (3.141, 9.424)

Ans

10. The least integer value of a for which the the series $\sum_{n=1}^{\infty} \frac{1}{n^{a-27}}$ converges is

(A) 26 (B) 27 (C) 28 (D) 29 (E) 30

Ans

11. The graph of the function represented by the Maclaurin series $x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \frac{x^6}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$ intersects the graph of $y = 1 + x^2$ at the point where $x =$

(A) 0.718 (B) 0.738 (C) 0.758 (D) 0.778 (E) 0.798

Ans

12. A particle is traveling along the circle $x^2 + y^2 = 4$. When it is at the point $(1, \sqrt{3})$, $\frac{dx}{dt} = 2$. Find $\frac{dy}{dt}$ at this instant.

(A) $-\frac{2}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) 0 (D) $\frac{1}{\sqrt{3}}$ (E) $\frac{2}{\sqrt{3}}$

Ans

---

13. Suppose interest on money in a bank account accumulates at an annual rate of 4% per year compounded continuously. If the balance $B = B(t)$ in the account satisfies the equation $\frac{dB}{dt} = .04B$, then approximately how much money should be invested today so that 5 years from now it would be worth \$4000?

(A) \$3600
(B) \$3300
(C) \$3000
(D) \$2700
(E) \$2400

Ans

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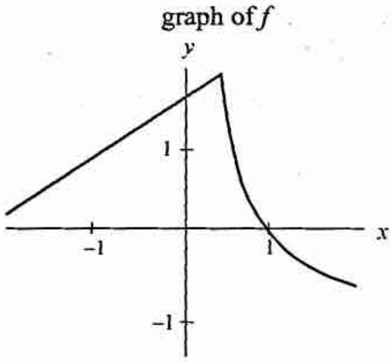
14. A graph of the function f, is shown in the figure. Which of the following are true?



(A) $f''(1) < f'(1) < f(1)$
(B) $f'(1) < f(1) < f''(1)$
(C) $f(1) < f'(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f(1) < f''(1) < f'(1)$

Ans

15. If $\int f(x)\cdot\sec^2 x\,dx = f(x)\cdot\tan x - \int 9x^2\cdot\tan x\,dx$, then $f(x)$ could be

(A) $x^3\cdot\sec^2 x$ (B) $x^3\cdot\tan x$ (C) $3x^3$ (D) $3x^2$ (E) $6x^3$

Ans

16. If the substitution $\sqrt{x} = u-1$ is made in $\int_1^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$, the resulting integral is

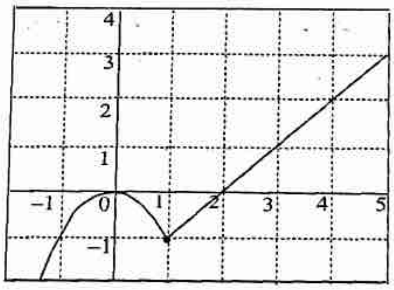
(A) $\int_1^4 \frac{2}{u}\,du$ (B) $\int_1^4 \frac{1}{u(u-1)}\,du$ (C) $\int_1^4 \frac{1}{u}\,du$

(D) $\int_2^3 \frac{2}{u}\,du$ (E) $2\int_2^3 \frac{1}{u(u-1)}\,du$

Ans

17. If the graph of f shown in the figure has a horizontal tangent at $x = 0$, which of the following statements are true.

graph of f



I. $\lim_{x\to 2} \frac{f(x)}{\sin(x-2)} = 1$

II. $\lim_{x\to 1} \frac{f(x-1)}{f(x+1)} = 1$

III. $\lim_{x\to 2} \frac{\left[f(x)\right]^2}{(x-2)^2} = 1$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, III

Ans

EXAM VI
CALCULUS BC
SECTION II, PART A
Time–30 minutes
Number of problems–2

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

1. At an electrical substation readings of the rate at which power is being used were recorded at 3 hour intervals over a 24-hour period and listed in the following table. The rate of power usage is in kilowatts per hour and is given by a differentiable function P at time t.

t	0	3	6	9	12	15	18	21	24
$P(t)$	1245	1268	1321	1316	1393	1369	1369	1451	1428

(a) Using a midpoint approximation with four equally spaced intervals, approximate $\int_0^{24} P(t)\,dt$.

Using correct units, explain the meaning of your answer in terms of power usage.

(b) Estimate how fast the rate of change of power usage is increasing at time $t = 12$. Show the computation that leads to your answer. Indicate units of measure.

(c) Assume that the function f, given by $f(t) = 1245 + 10te^{0.25\cos t}$, is an accurate model of the rate of power usage at time t, where t is measured in hours and $f(t)$ is in kilowatts per hour. Use $f(t)$ to approximate the average rate of power usage during the 24-hour time period. Undicate units of measure.

2. Let g be the function given by $g(x) = \int_0^x \cos(e^{t/2})\,dt$ for $-1 \le x \le 4$.

a) Find $g(1/2)$.

b) Find the instantaneous rate of change of g, with respect to x, at $x = 0$.

c) Find all values of x on the interval $(-1, 4)$ at which g has a relative maximum. Justify your answer.

d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-1, 4)$. Justify your answer.

e) Find the absolute maximum of g on the closed interval $[0, 4]$. Justify your answer.

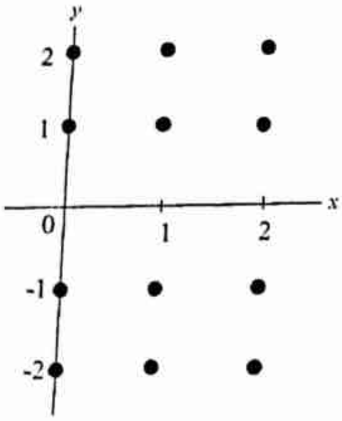
Time - 60 minutes
Number of problems - 4

A graphing calculator may NOT be used on this part of the examination.

- During the timed portion for part B, you may go back and continue to work on the problems

3. A function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = \frac{x-1}{y}$, with initial condition $f(0) = -2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Sketch the solution curve that passes through the point (0, −2).

(c) Find $f''(0)$.

(d) Find the particular solution to the differential equation that satisfies the initial condition $f(0) = -2$.

4. A particle with coordinates $(x(t), y(t))$ moves along a curve in the first quadrant so that

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t+1}} \quad \text{and} \quad \frac{dy}{dt} = \frac{1}{(t+1)^2} \qquad \text{for } t \geq 0.$$

(a) Find the coordinates of the particle in terms of t if, when $t = 0$, $x = 1$ and $y = 0$.

(b) Write an equation expressing y in terms of x.

(c) Find the average rate of change of y with respect to x as t varies from $t = 3$ to $t = 15$.

(d) Find the instantaneous rate of change of y with respect to x when $t = 8$.

5. The part of the curve of $y^2 - y + e^x = \cos x$ that is near the point (0, 1) defines y as a function of x implicitly.

(a) Find $\frac{dy}{dx}$.

(b) Find an equation of the line tangent to curve at the point (0, 1).

(c) Find $\frac{d^2y}{dx^2}$ at the point (0, 1).

(d) Use the tangent line approximation in part (b) to estimate the value of y for the point on the curve near (0, 1) where $x = 0.5$.

(e) Does the tangent line approximation in part (d) yield a larger or smaller value than the actual y-value. Explain.

6. Let f be the function defined by $f(x) = \frac{1}{4+x^2}$.

(a) Write the first four terms and general term for the power series expansion of f about $x = 0$.

(b) Determine the radius of convergence of the series found in part (a).

(c) Given the function g defined by $g(x) = \int_0^x \frac{1}{4+t^2}\,dt$, find the Taylor expansion of g about $x = 0$.

(d) Using your answer in part (c), show that $\frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + \cdots = \frac{\pi}{8}$

Answers

EXAM I

Part A				Part B	
1.	D	18.	D	1.	C
2.	C	19.	A	2.	C
3.	B	20.	D	3.	D
4.	C	21.	C	4.	B
5.	D	22.	C	5.	D
6.	E	23.	D	6.	E
7.	C	24.	C	7.	B
8.	A	25.	D	8.	D
9.	A	26.	E	9.	A
10.	D	27.	C	10.	B
11.	C	28.	B	11.	E
12.	B			12.	B
13.	E			13.	A
14.	B			14.	B
15.	E			15.	C
16.	D			16.	B
17.	E			17.	B

EXAM II

Part A				Part B	
1.	A	18.	A	1.	C
2.	C	19.	C	2.	C
3.	A	20.	C	3.	D
4.	A	21.	D	4.	C
5.	C	22.	A	5.	C
6.	C	23.	A	6.	C
7.	A	24.	E	7.	E
8.	D	25.	A	8.	B
9.	B	26.	B	9.	E
10.	D	27.	E	10.	A
11.	A	28.	E	11.	D
12.	A			12.	B
13.	D			13.	D
14.	D			14.	B
15.	C			15.	B
16.	E			16.	E
17.	B			17.	B

EXAM I Section II Part A

1. (a) ≈ 0.902 sec

(b) $x = 5, y = 12,\ t = 0.247,\ \dfrac{dx}{dt} = 18.827$ ft/sec

2. (a) $g'(0) = 1$ (b) $g'(1) > 0 \Rightarrow g$ is increasing

(c) $g''(0) \approx 7$ (d) $g''(1) > 0 \Rightarrow g$ – graph concave up

EXAM I Section II Part B

3. (a) $2 \cdot \dfrac{1}{2}\displaystyle\int_0^{\pi}(2-\cos\theta)^2\,d\theta$

(b) $dx/d\theta = -2\sin\theta + 2(\cos\theta)(\sin\theta)$

$dy/d\theta = 2\cos\theta - \cos^2\theta + \sin^2\theta$

(c) $\dfrac{dy}{dx} = \dfrac{2\cos\theta - \cos^2\theta + \sin^2\theta}{-2\sin\theta + 2(\cos\theta)(\sin\theta)}$

4. (a) all x (b) $1 - \dfrac{x^2}{2!} - \dfrac{x^4}{4!} - \cdots + \dfrac{(-1)^n x^{2n}}{(2n)!}$

(c) $g'(x) = \cos x,\ f(x) = \begin{cases} \dfrac{1-\cos x}{x^2} & \text{if } x \neq 0 \\ 1/2 & \text{if } x = 0 \end{cases}$

(d) $g(x) = \sin x + 3$

5. (a) $-\dfrac{2}{\sqrt{t+1}}$ (b) $x + y = \dfrac{5}{4}$

(c) $\displaystyle\int_0^1 \sqrt{\frac{1}{4(t+1)^3} + \frac{1}{(t+1)^4}}\,dt$ (d) $y = 1 - x^2, 0 < x \le 1$

6. (a) $P(t) = 1200 - 900e^{-kt}$

(b) $k = -\dfrac{\ln(2/3)}{4} = \dfrac{1}{4}\ln\dfrac{3}{2}$

(c) $\lim\limits_{t\to\infty} P(t) = 1200$

EXAM II Section II Part A

1. (a) $v(t) = 1 - t(\cos t) - (\ln t)(\sin t)$

(b) $\max|v(t)| = 5.896$ when $t = 6.700$

(c) $5.204 < t < 7.987$ (d) 21.461

2. (a) decreasing at 0.0375 ft per ft

(b) 27.772 cu ft (c) $w = 1.155$ ft, $h = 1.633$ ft

EXAM II Section II Part B

3. (a) $2 + \ln 3$ (b) $4\pi k\left(1 + \dfrac{k}{6}\right)$

(c) $\pi\displaystyle\int_1^3 \left[x + \frac{k}{x} + 2\right]^2 - [x+2]^2\,dx$

4. (a) -3 (b) 4 (c) 33

5. (a) $\dfrac{1}{k}$ (b) $\left(\dfrac{1}{k}, \dfrac{1}{ke}\right)$ is a rel max

(c) $x > \dfrac{2}{k}$ (d) $y = 0$

6. (a) $f'(0) = 1, f''(0) = -1, f'''(0) = 2$

(b) $x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots + \dfrac{(-1)^{n+1}x^n}{n} + \cdots$

(c) 1

EXAM III

Part A				Part B	
1.	C	18.	D	1.	A
2.	C	19.	D	2.	B
3.	C	20.	C	3.	D
4.	D	21.	C	4.	D
5.	D	22.	D	5.	C
6.	E	23.	C	6.	B
7.	E	24.	B	7.	A
8.	C	25.	C	8.	D
9.	C	26.	D	9.	B
10.	C	27.	D	10.	D
11.	E	28.	A	11.	C
12.	E			12.	C
13.	C			13.	A
14.	A			14.	C
15.	E			15.	C
16.	D			16.	D
17.	E			17.	E

EXAM III Section II Part A

1. (*a*) -3.75

   (*b*) $y-2.6 = -3.75(x-1.4)$

   (*c*) $f''(1.4) > 0$

   (*d*) $.04[f(1.2)+f(1.6)] = 2.2$

2. (a) $(\sqrt{2},1)$, $(-\sqrt{2},-1)$

   (b) product of slopes $= -1$ (c) 1.737

EXAM III Section II Part B

3. (a) 10 (b) $x = -3, 1, 3$

   (c) $(-6,-4)$ and $(1,3)$

   (*d*) $H'(4) = f'[3]\cdot 5 = 2\cdot 5 = 10$

4. (a) diverges (b) $-\frac{1}{3} \le x < \frac{1}{3}$

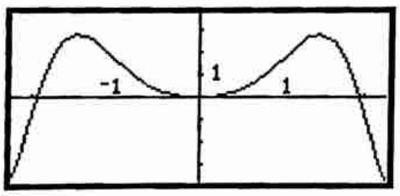
5. (a) $v(t) = \frac{-6}{\sqrt{2t+1}} + 3$ (b) 1.5

   (c) $s(t) = -6\sqrt{2t+1} + 3t + 15$ (d) 3

6. (a) $h(-x) = h(x)$ (b) $-\sqrt{5} \le x \le \sqrt{5}$

   (c) -12 (d) $\pm\sqrt{2}$

   (e)



EXAM IV

Part A				Part B	
1.	D	18.	B	1.	D
2.	B	19.	D	2.	D
3.	C	20.	E	3.	C
4.	B	21.	D	4.	C
5.	D	22.	E	5.	C
6.	C	23.	A	6.	A
7.	C	24.	B	7.	C
8.	E	25.	B	8.	E
9.	D	26.	D	9.	B
10.	D	27.	D	10.	C
11.	C	28.	B	11.	D
12.	D			12.	D
13.	B			13.	C
14.	C			14.	C
15.	E			15.	B
16.	B			16.	D
17.	C			17.	D

EXAM IV Section II Part A

1. (a) rel min at $x = -0.364$ and $x = 1.977$
   rel max at $x = 0.487$

   (b) $-0.364 < x < 0.487$ and $x > 1.977$

   (c) $0.043 < x < 1.092$ (d) 1.097 units^2

2. (a) $\frac{2}{3}\cot t$ (b) $2x - 3y + 5 + 6\sqrt{2} = 0$

   (c) 3.757

EXAM IV Section II Part B

3. (a) 0.4 ft / sec^2

   (b) 1300 feet

   (c) Car A is traveling faster

4. (a) $1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{n!}$

   (b) converges for all x (radius is infinite)

5. (a) $y = \left(C - \frac{kt}{2}\right)^2$ (b) $y(t) = \left(3 - \frac{t}{20}\right)^2$

   (c) 40 min

6. (a) 1.5 (b) $2 < x < 2.5$

   (c) $G(3) > 0$ and $G(4) < 0$ so by the Intermediate Value Theorem, G has a zero in $[3,4]$. $G'(x) < 0$ on $[3,4]$, so G is monotone, decreasing and has only one zero in $[3,4]$.

   (d) $y-1.5 = -1(x-3)$

Answers

EXAM V

Part A				Part B	
1.	C	18.	B	1.	D
2.	C	19.	B	2.	B
3.	C	20.	B	3.	B
4.	C	21.	D	4.	E
5.	B	22.	E	5.	B
6.	C	23.	B	6.	B
7.	D	24.	D	7.	C
8.	E	25.	B	8.	E
9.	E	26.	A	9.	A
10.	C	27.	D	10.	A
11.	D	28.	E	11.	D
12.	A			12.	E
13.	C			13.	D
14.	E			14.	D
15.	D			15.	A
16.	B			16.	B
17.	A			17.	D

EXAM VI

Part A				Part B	
1.	A	18.	C	1.	E
2.	E	19.	B	2.	B
3.	B	20.	D	3.	C
4.	E	21.	D	4.	D
5.	E	22.	C	5.	B
6.	D	23.	B	6.	C
7.	D	24.	D	7.	B
8.	C	25.	C	8.	D
9.	D	26.	C	9.	B
10.	A	27.	C	10.	D
11.	C	28.	A	11.	B
12.	C			12.	A
13.	B			13.	B
14.	D			14.	B
15.	B			15.	C
16.	A			16.	D
17.	D			17.	D

EXAM V Section II Part A

1. (a) $y - e^a = e^a(x-a)$ $R = (a-1, 0)$ (b) $A = \frac{1}{2}e^a|a-1|$

   (c) Area max when $a = 0$. $A'(0) = 0$ and $A''(0) < 0$

   $A(-1) = 0.368$, $A(0) = 0.5$, $A(1) = 0$

2. (a) $R_n = \sum_{k=1}^{n}\left(1\cdot\Delta x_k\cdot\frac{1}{1+x_k}\right)(x_k)^2$ When $n = 4$, $\Delta x = 1$, $x_1 = 0$

   $R_4 = (1\cdot1\cdot1)(0)^2 + (1\cdot1\cdot\frac{1}{2})(1)^2 + (1\cdot1\cdot\frac{1}{3})(2)^2 + (1\cdot1\cdot\frac{1}{4})(3)^2 = \frac{49}{12}$

   (b) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1\cdot\Delta x_k\cdot\frac{1}{1+x_k}\right)(x_k)^2 = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{(x_k)^2}{1+x_k}\Delta x = \int_0^4\frac{x^2}{1+x}dx$

   (c) $\int_0^4\frac{x^2}{1+x}\,dx = 5.609$ grams

EXAM V Section II Part B

3. (a) $T_3(x) = \frac{1}{3}(x-2) + \frac{1}{18}(x-2)^2 + \frac{1}{81}(x-1)^3$

   (b) Interval of convergence is all x for which $-1 \le x < 5$.

4. (a) $\frac{dy}{dx} = \frac{2x}{3y^2+2y-5}$ (b) $y - 0 = -\frac{4}{5}(x-2)$

   (c) vertical $x = \pm1$, or $\pm\sqrt{\frac{283}{27}}$ horizontal $x = 0$

5. (a) $x = 10$ (b) $5 < x < 12.5$

   (c) critical points $(10, 60)$ and $(15, 47.5)$
   inflection points $(5, 35)$ and $(12.5, 53.75)$

6. (b) $|y| = 2e^{x^2/4}$ (c) $y = 2e$

   (d) $y = 3$; exact: $y(2) = 5.437$

EXAM VI Section II Part A

1. (a) $6(1268 + 1316 + 1369 + 1451) = 32{,}424$ K

   (b) $\left(\frac{1369-1316}{6}\right) = 8.833$ K/hr^2

   (c) $\frac{1}{24}\int_0^{24}(1245+10te^{.25\cos t})\,dt = 1364.478$ K/hr

2. (a) $g(1/2) = 0.210$ (b) $g'(0) = 0.540$

   (c) $x = 0.903$ (d) $x = 2.290$ or $x = 3.676$

   (e) 0.269

EXAM VI Section II Part B

3. c) $-\frac{3}{8}$ d) $y = -\sqrt{x^2-2x+4}$.

4. (a) $x(t) = \sqrt{t+1}$; $y(t) = 1 - \frac{1}{t+1}$ (b) $y = 1 - \frac{1}{x^2}$

   (c) $\frac{3}{32}$ (d) $\frac{2}{27}$

5. (a) $\frac{dy}{dx} = \frac{-e^x - \sin x}{2y-1}$ (b) $y = 1 - x$

   (c) At $(0,1)$, $\frac{d^2y}{dx^2} = -4$ (d) $y(0.5) \approx 0.5$ (e) larger

6. (a) $\frac{1}{4} - \frac{x^2}{16} + \frac{x^4}{64} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{2^{2n+2}}$ (b) R = 2

   (c) $\frac{1}{4}x - \frac{x^3}{48} + \frac{x^5}{320} - \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)2^{2n+2}}$

   (d) Let $x = 2$